Other uses of integer variables

(i) Suppose x can only take one of a finite set $\{v_1, v_2, \dots, v_m\}$ of values.

$$x = v_1 x_1 + v_2 x_2 + \dots + v_m x_m$$

$$x_1 + x_2 + \dots + x_m = 1$$

$$x_i = 0 \text{ or } 1, \; i = 1, 2, \dots, m$$

(ii) $x \le 1$ or $x \ge 2$

Assume $0 \le x \le M$ $\quad M \ge 2$

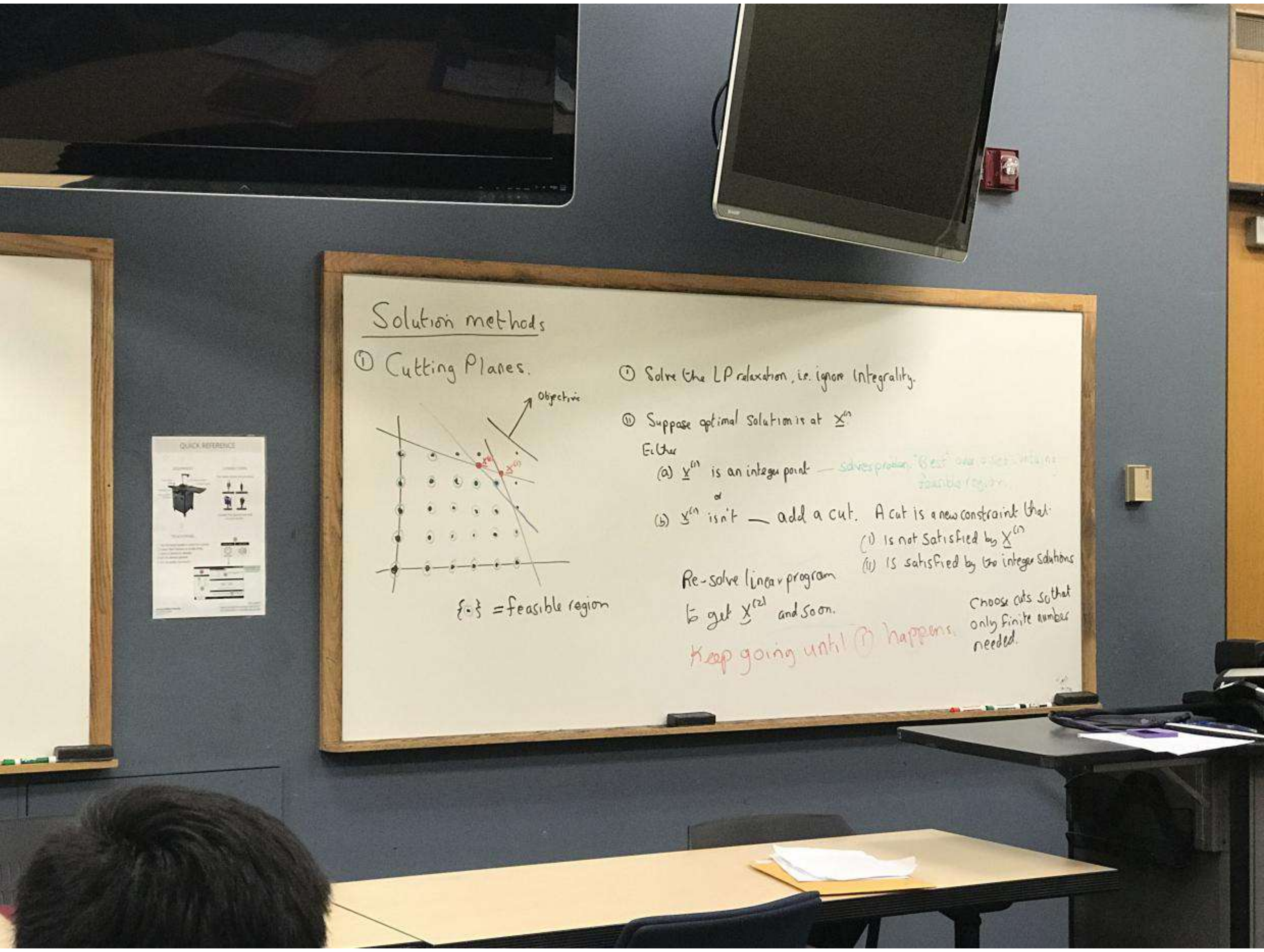
Introduce $\delta = 0$ or 1

$$x \ge 0$$

$$x \le 1 + (M-1)\delta$$

$$x \ge 2 + M(\delta - 1)$$

$$\delta = 0 \text{ or } 1$$


Solution methods
① Cutting Planes.
Objective
{·} = feasible region
① Solve the LP relaxation, i.e. ignore Integrality.
② Suppose optimal solution is at $\underline{x}^{(1)}$
Either
(a) $\underline{x}^{(1)}$ is an integer point
or
(b) $\underline{x}^{(1)}$ isn't — add a cut. A cut is a new constraint that:
(i) Is not satisfied by $\underline{x}^{(1)}$
(ii) Is satisfied by the integer solutions
Re-solve linear program
to get $\underline{x}^{(2)}$ and so on.
Keep going until ① happens.
Choose cuts so that only finite number needed.

Gomory Cuts for the pure problem: all variables must be integer.

Suppe we solve the <u>LP relaxation</u> and we find a basic variable x_i that is not an integer.

In the tableau we have

$$(*) \quad x_i + \sum_{j \text{ nonbasic}} b_{ij} x_j = b_{i0}$$

NOT AN INTEGER

Find an inequality that is satisfied by all non-negative integer solutions to $(*)$, but not by putting $x_j = 0$, j nonbasic

More generally, suppose we have

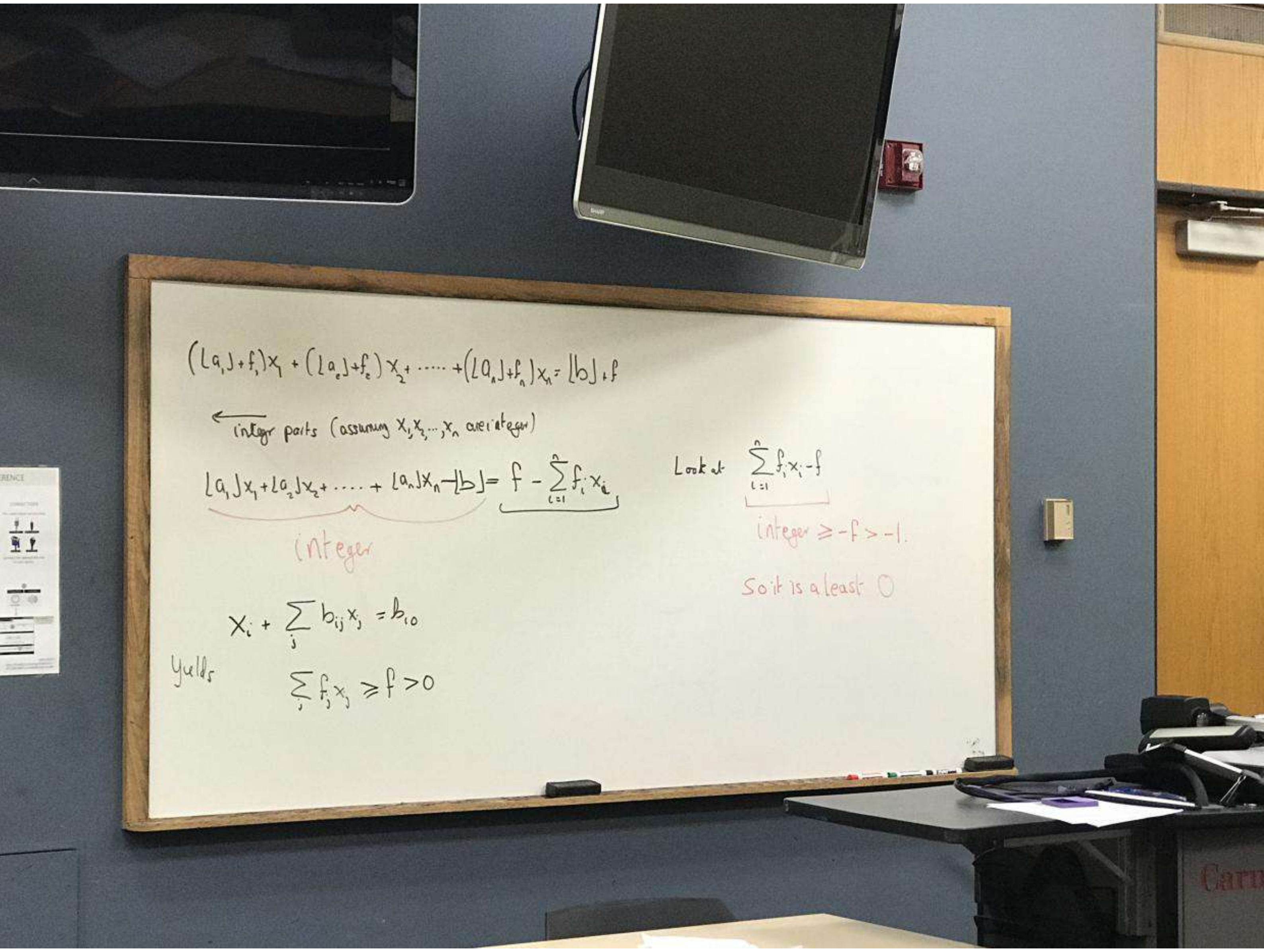
$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b$$

Write $a_i = \lfloor a_i \rfloor + f_i$ $\qquad b = \lfloor b \rfloor + f$

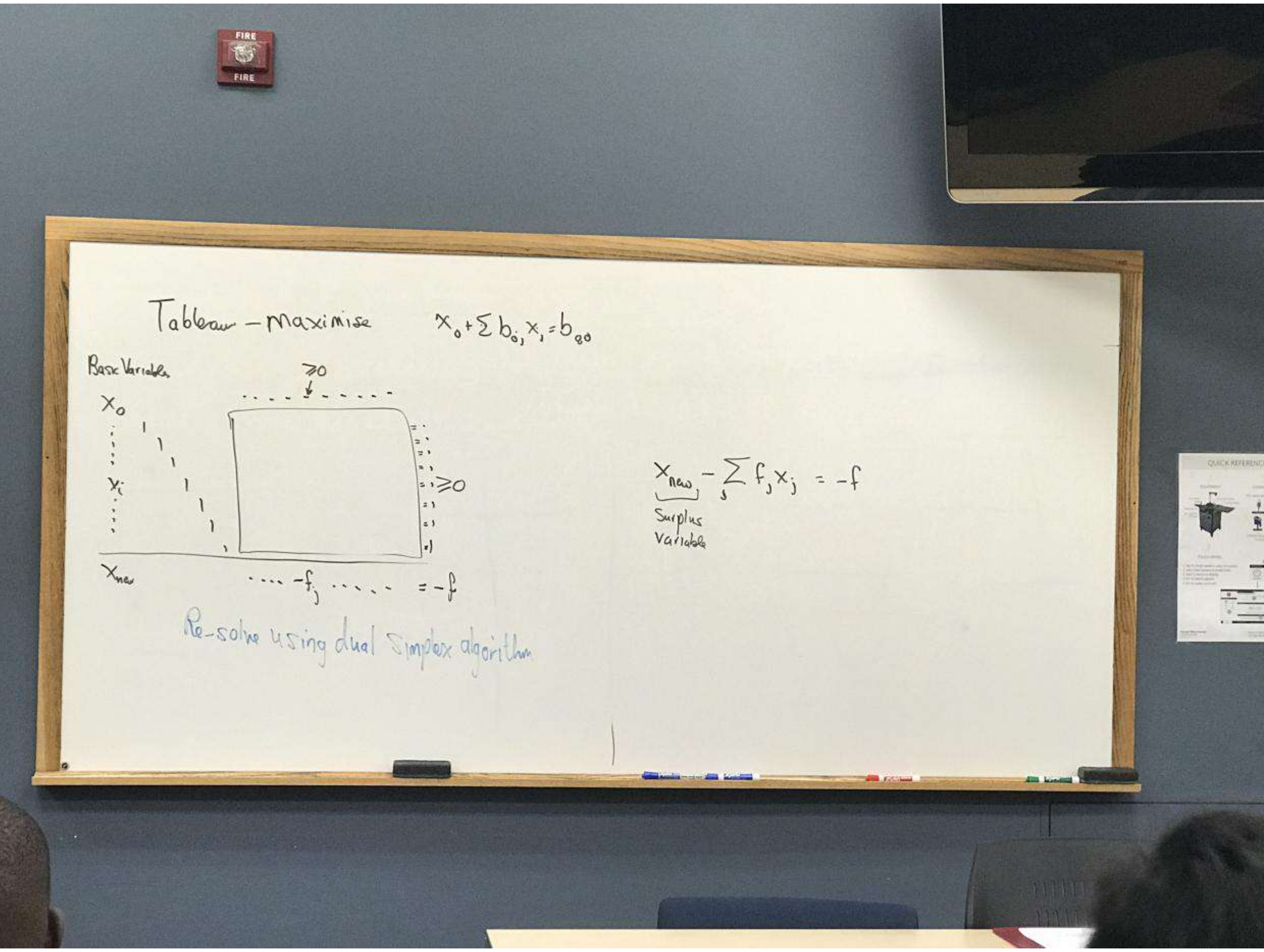
integer part $\qquad$ fractional part $\qquad f > 0$

$$9\tfrac{1}{4} = 9 + \tfrac{1}{4}$$

$$-9\tfrac{1}{4} = -10 + \tfrac{3}{4}$$



$(\lfloor a_1 \rfloor + f_1)x_1 + (\lfloor a_2 \rfloor + f_2)x_2 + \cdots + (\lfloor a_n \rfloor + f_n)x_n = \lfloor b \rfloor + f$
integr parts (assuming $x_1, x_2, \ldots, x_n$ are integer)
$\underbrace{\lfloor a_1 \rfloor x_1 + \lfloor a_2 \rfloor x_2 + \cdots + \lfloor a_n \rfloor x_n - \lfloor b \rfloor}_{\text{integer}} = \underbrace{f - \sum_{i=1}^{n} f_i x_i}$
Look at $\underbrace{\sum_{i=1}^{n} f_i x_i - f}$
integer $\geq -f > -1$.
So it is a least 0
$x_i + \sum_j b_{ij} x_j = b_{i0}$
yields
$\sum_j f_j x_j \geq f > 0$


FIRE
FIRE
Tableau – maximise
$x_0 + \sum b_{0j} x_j = b_{00}$
Basic Variables
≥ 0
x_0
x_i
≥ 0
x_{new}
$\cdots -f_j \cdots = -f$
Re-solve using dual simplex algorithm
$\underbrace{x_{new}}_{} - \sum_j f_j x_j = -f$
Surplus Variable