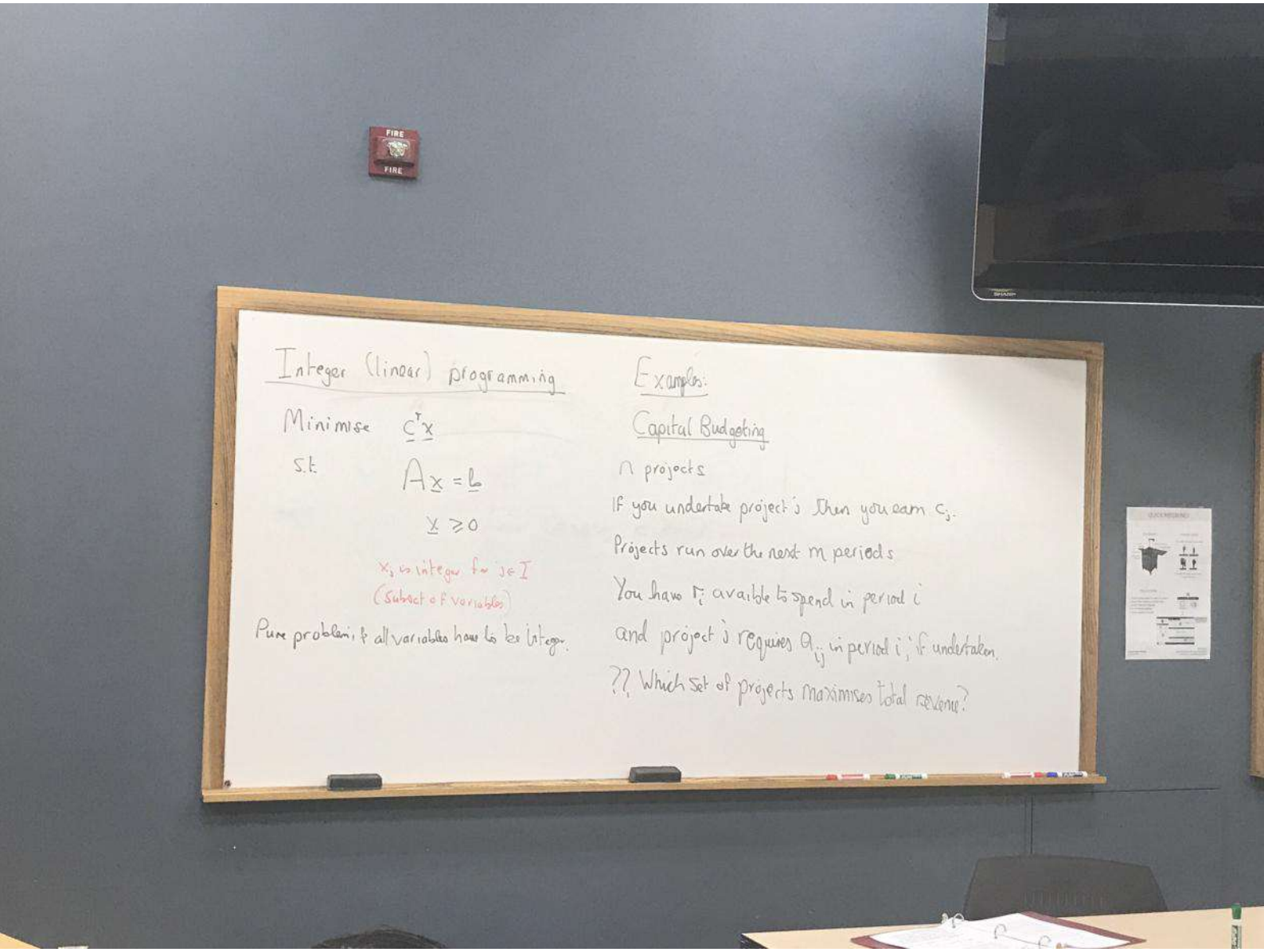
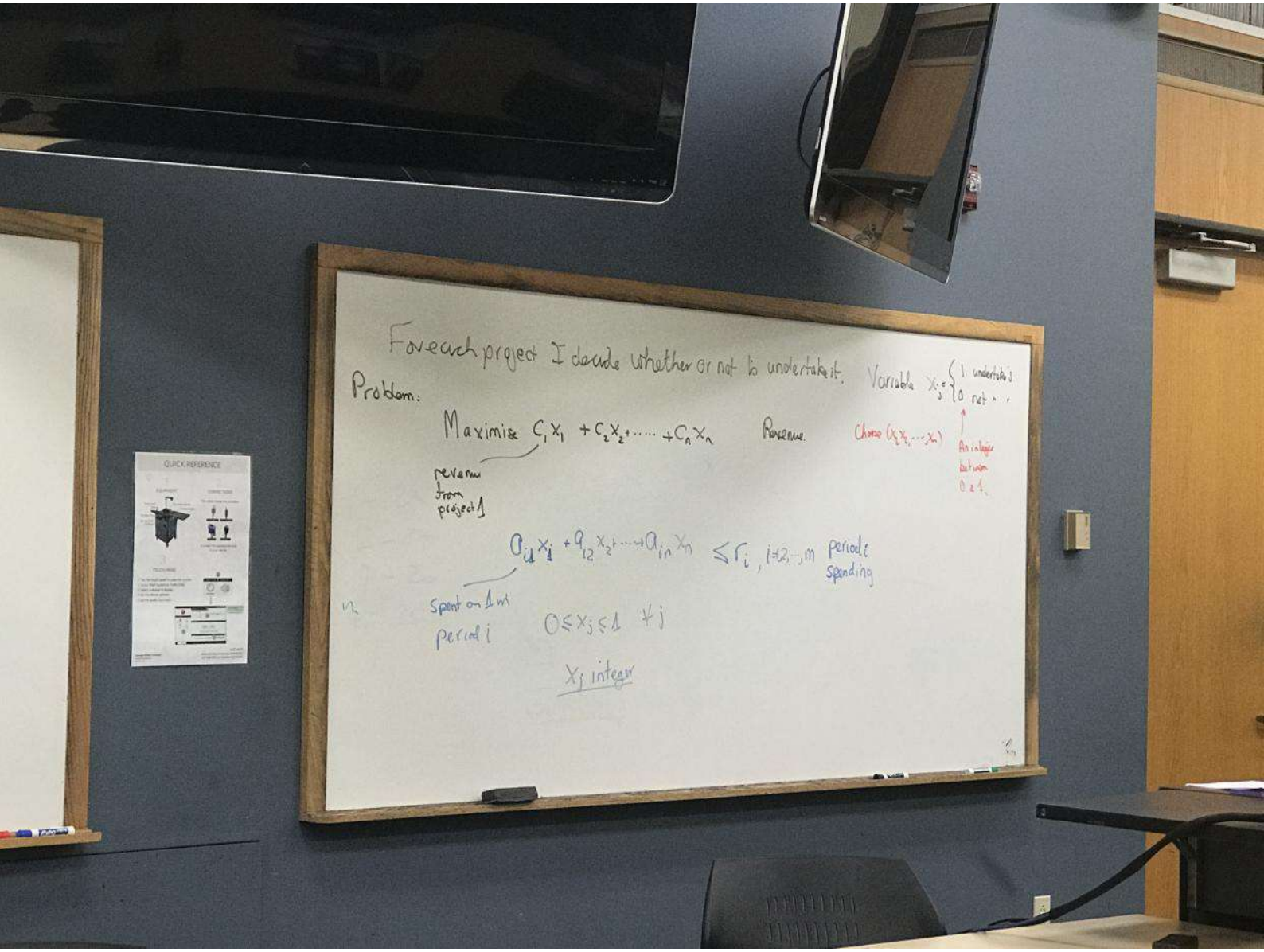
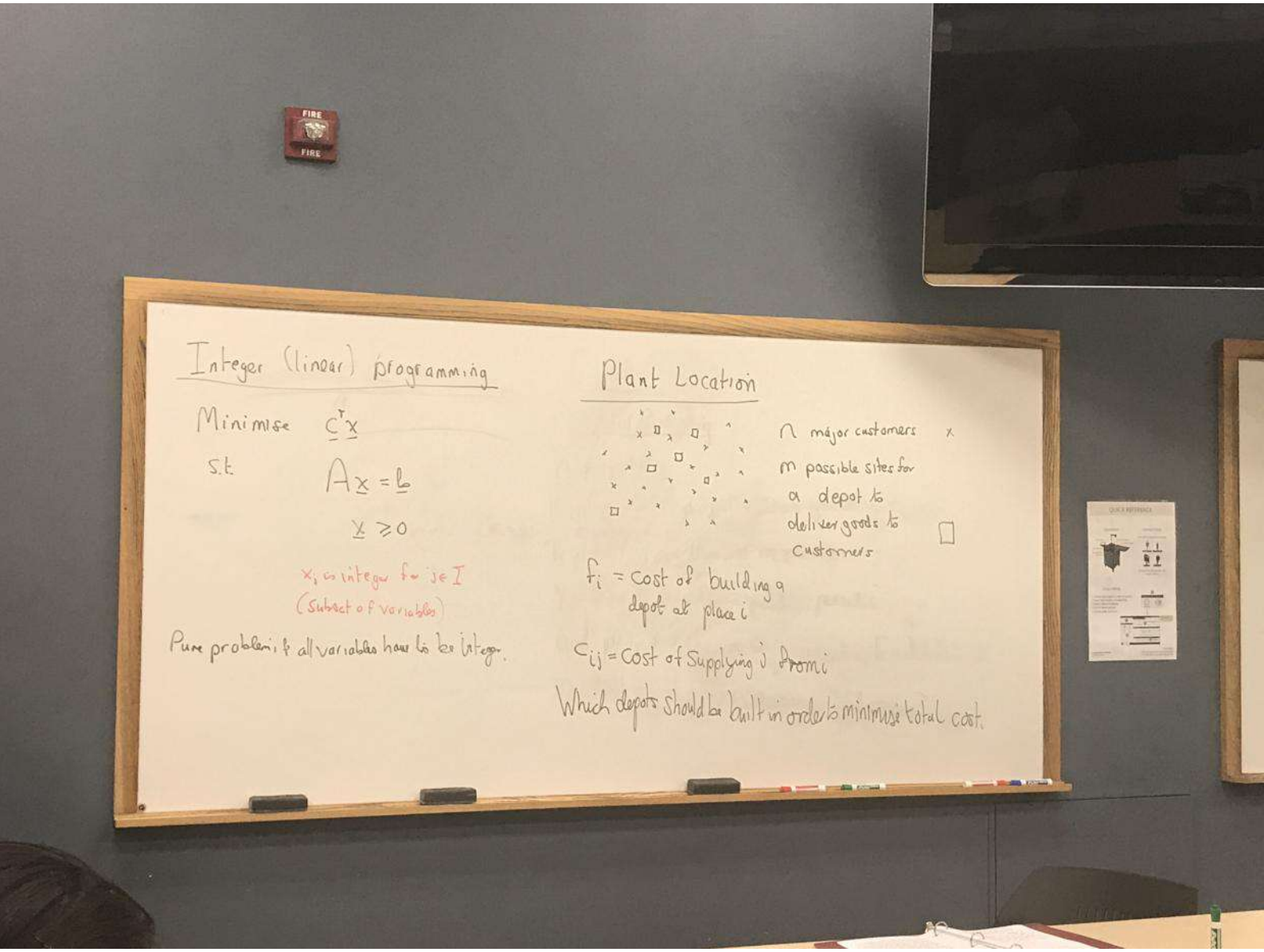

FIRE
FIRE
Integer (linear) programming
Minimise $\underline{c}^T\underline{x}$
s.t $A\underline{x} = \underline{b}$
$\underline{x} \geq 0$
x_j is integer for $j \in I$
(Subset of variables)
Pure problem: all variables have to be integer.
Examples:
Capital Budgeting
n projects
If you undertake project j then you earn c_j.
Projects run over the next m periods
You have r_i available to spend in period i
and project j requires a_{ij} in period i, if undertaken.
?? Which set of projects maximises total revenue?


For each project I decide whether or not to undertake it.
Variable $x_j = \begin{cases} 1 & \text{undertake } j \\ 0 & \text{not} \end{cases}$
Problem:
Maximise $c_1x_1 + c_2x_2 + \cdots + c_nx_n$
Revenue.
Choose $(x_1, x_2, \ldots, x_n)$
An integer between 0 & 1
revenue from project 1
$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \leq r_i$, $i = 1, 2, \ldots, m$
period i spending
Spent on 1 in period i
$0 \leq x_j \leq 1 \quad \forall j$
x_j integer
QUICK REFERENCE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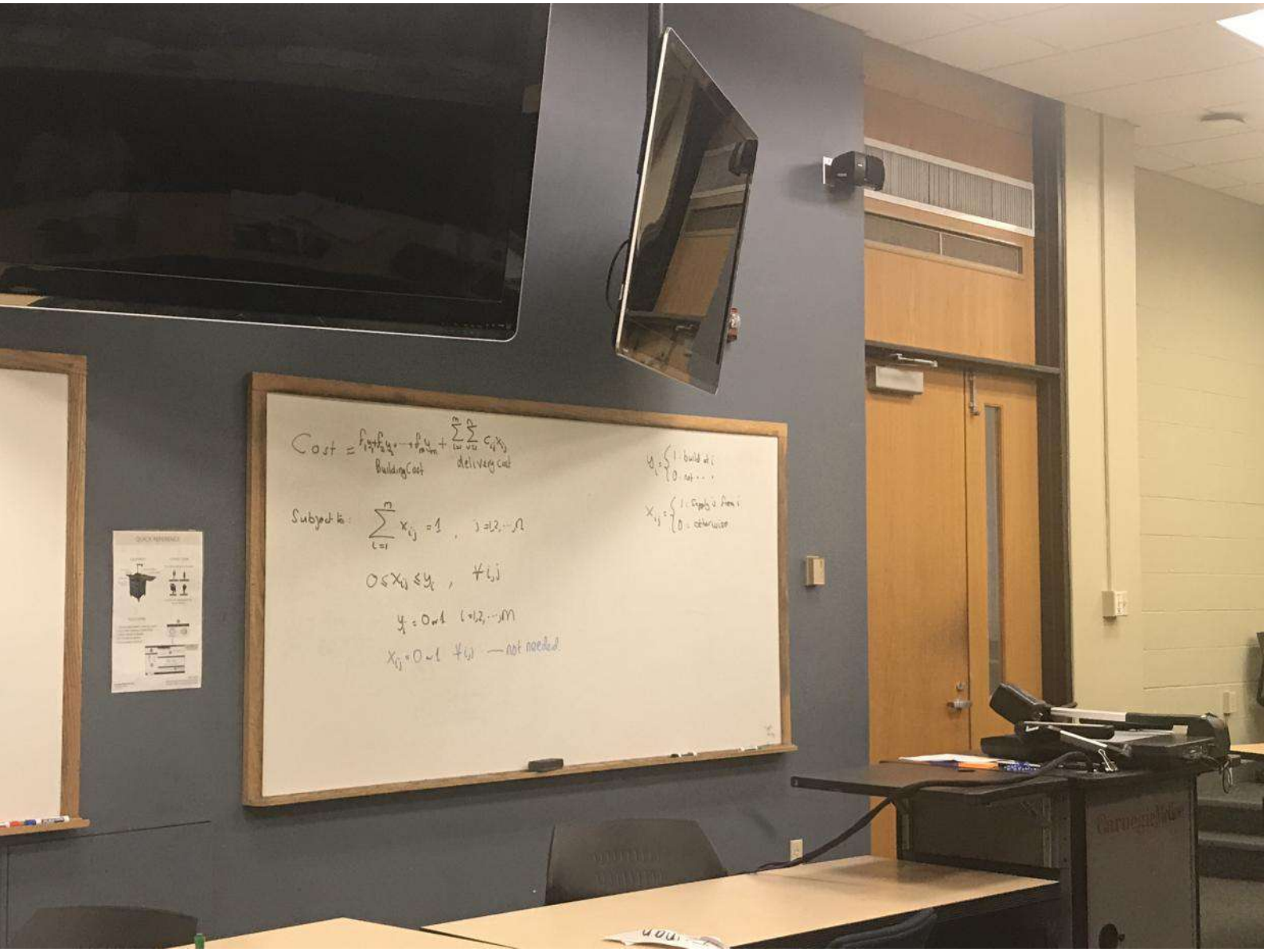
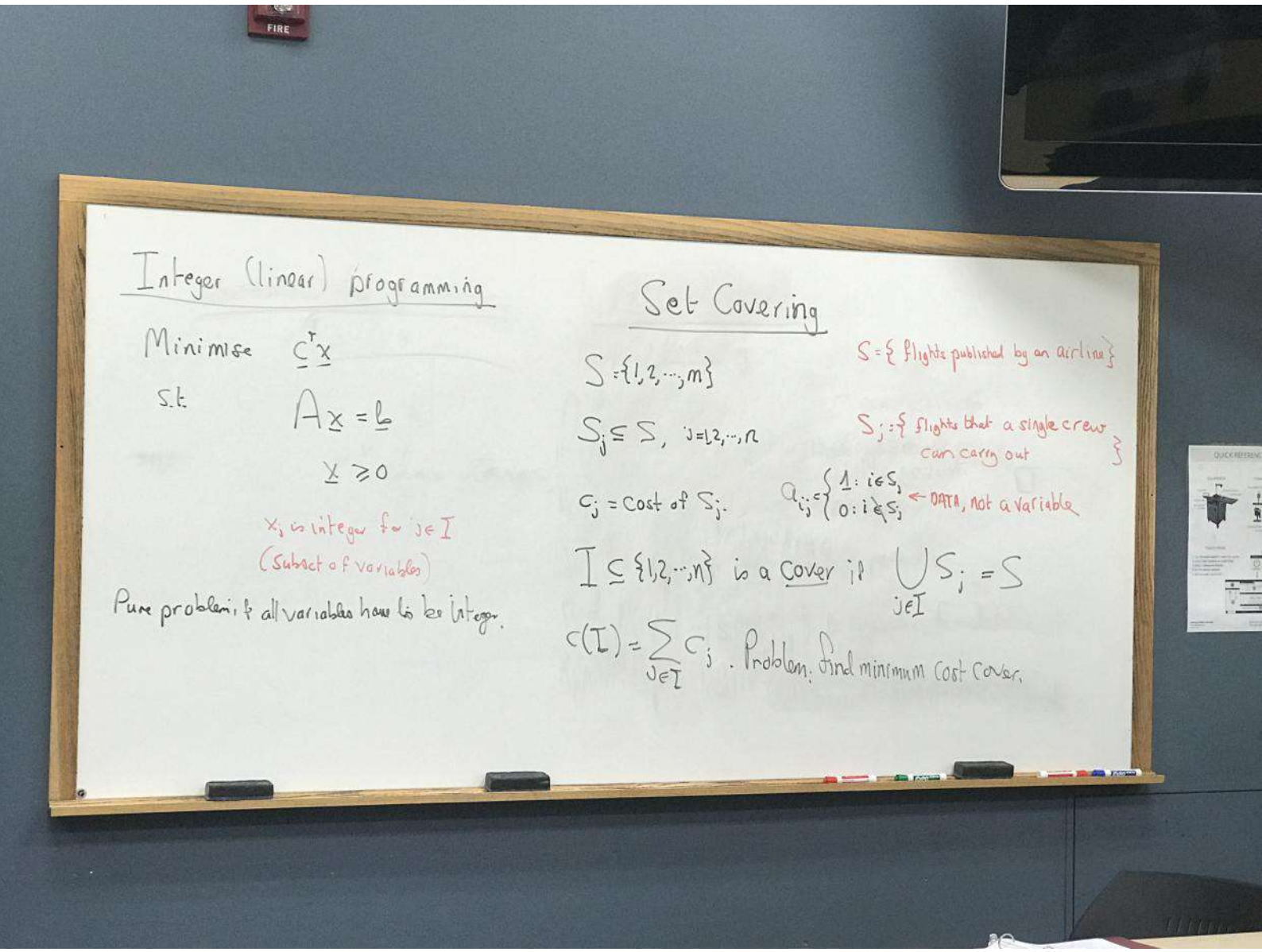
Integer (linear) programming
Minimise $\underline{c}^T\underline{x}$
s.t. $A\underline{x} = \underline{b}$
$\underline{x} \geq 0$
x_i is integer for $j \in I$
(Subset of variables)
Pure problem if all variables have to be integer.
Plant Location
n major customers
m possible sites for a depot to deliver goods to customers
f_i = cost of building a depot at place i
c_{ij} = cost of supplying j from i
Which depots should be built in order to minimise total cost.


Cost = $f_1 y_1 + f_2 y_2 + \cdots + f_m y_m + \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$
Building cost
delivery cost
$y_i = \begin{cases} 1 : \text{build at } i \\ 0 : \text{not} \cdots \end{cases}$
$x_{ij} = \begin{cases} 1 : \text{Supply } j \text{ from } i \\ 0 : \text{otherwise} \end{cases}$
Subject to: $\sum_{i=1}^{m} x_{ij} = 1, \quad j = 1, 2, \cdots n$
$0 \le x_{ij} \le y_i, \quad \forall i, j$
$y_i = 0 \text{ or } 1 \quad i = 1, 2, \cdots m$
$x_{ij} = 0 \text{ or } 1 \quad \forall i, j$ — not needed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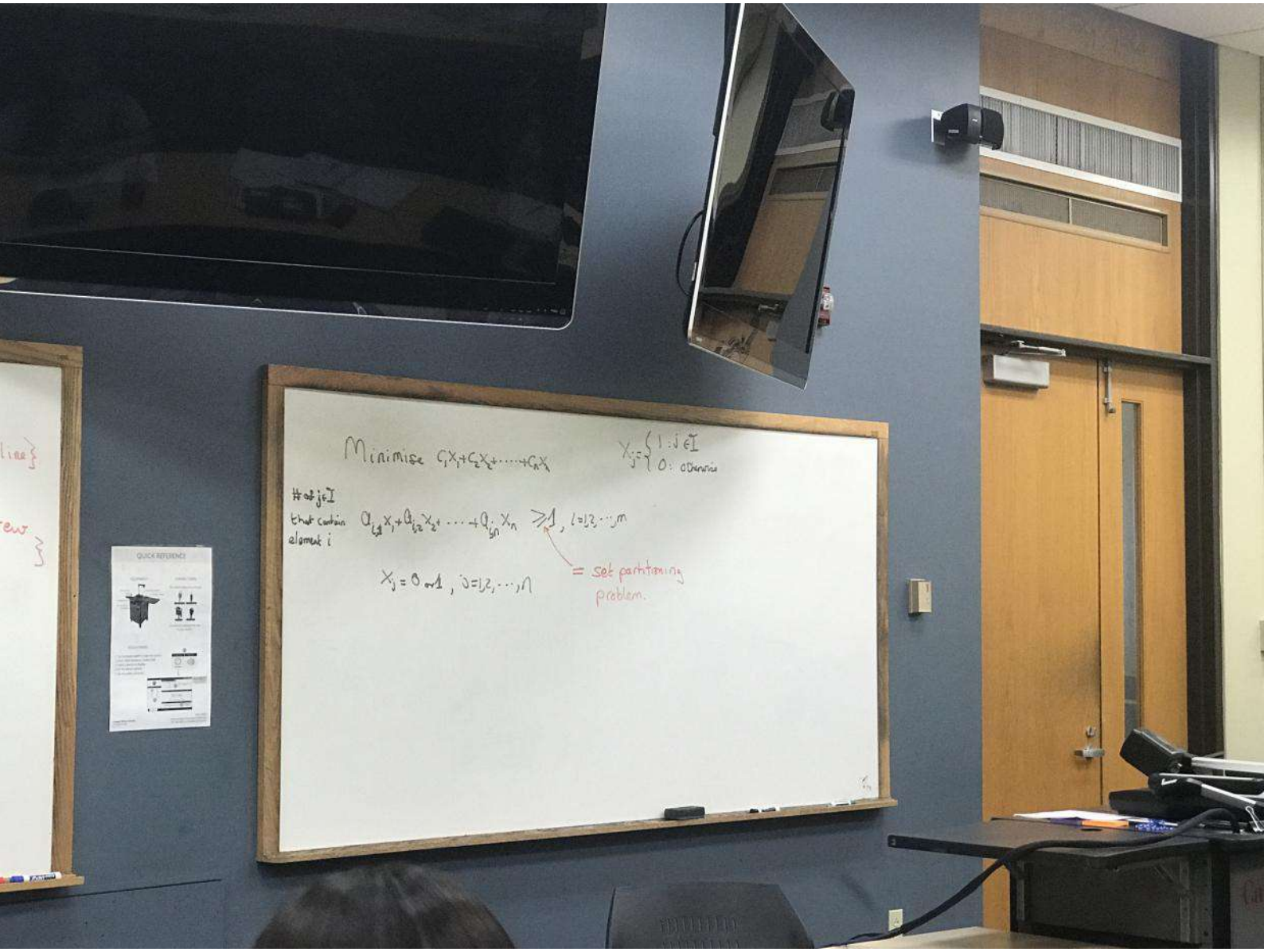
Integer (linear) programming
Minimise $\underline{c}^T\underline{x}$
s.t. $A\underline{x} = \underline{b}$
$\underline{x} \geq 0$
x_j is integer for $j \in I$
(Subset of variables)
Pure problem: if all variables have to be integer.
Set Covering
$S = \{1, 2, \cdots, m\}$
$S_j \subseteq S$, $j = 1, 2, \cdots, n$
$S = \{$ flights published by an airline $\}$
$S_j = \{$ flights that a single crew can carry out $\}$
c_j = cost of S_j.
$a_{ij} = \begin{cases} 1 : i \in S_j \\ 0 : i \notin S_j \end{cases}$ ← DATA, not a variable
$I \subseteq \{1, 2, \cdots, n\}$ is a cover if $\bigcup_{j \in I} S_j = S$
$c(I) = \sum_{j \in I} c_j$. Problem: find minimum cost cover.


Minimise $c_1x_1+c_2x_2+\cdots+c_nx_n$
$x_j = \begin{cases} 1 : j \in I \\ 0 : \text{otherwise} \end{cases}$
of $j \in I$ that contain element i
$a_{i1}x_1+a_{i2}x_2+\cdots+a_{in}x_n \geq 1, \quad i=1,2,\cdots,m$
$x_j = 0 \text{ or } 1, \quad j=1,2,\cdots,n$
= set partitioning problem.