Minimise $f(\underline{x})$, subject to $\underline{x} \in S$

Convex Program: f, S are convex.

Thm 2.1

If f and S are convex, then if $\underline{x}^*$ is a local minimum, it is also a global minimum.

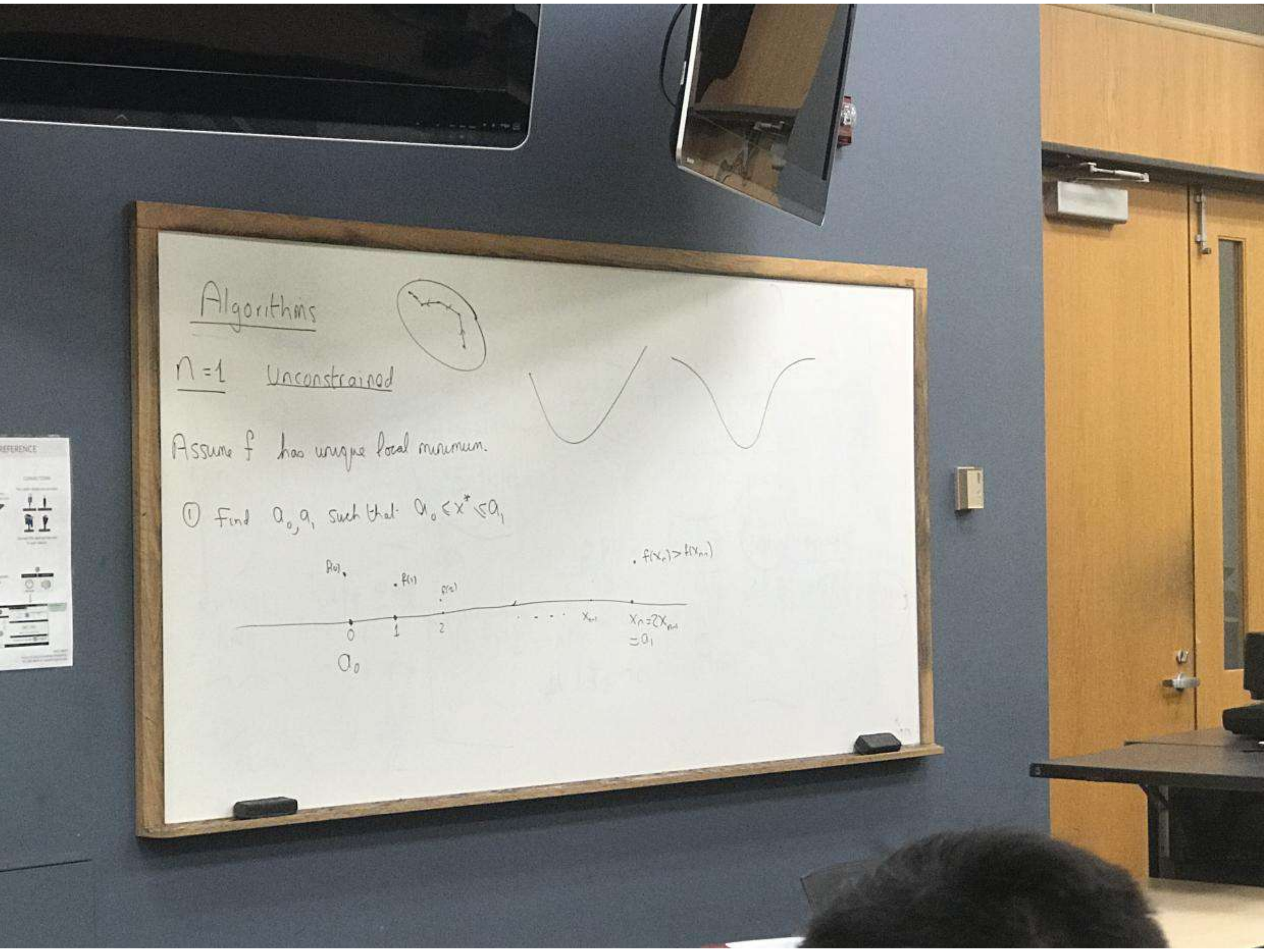
$f(\underline{x}^*)$ is minimum in $B \cap S$

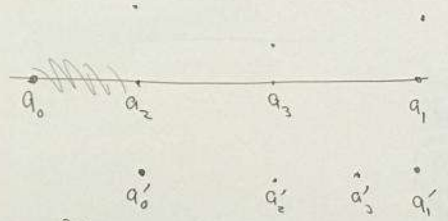
$\underline{z} = \lambda \underline{x}^* + (1-\lambda)\underline{x} \quad 0 < \lambda < 1$

$$f(\underline{x}^*) \le f(\underline{z}) \le \lambda f(\underline{x}^*) + (1-\lambda) f(\underline{x})$$

$$(1-\lambda) f(\underline{x}^*) \le (1-\lambda) f(\underline{x})$$

$$f(\underline{x}^*) \le f(\underline{x}).$$


Algorithms
n=1 Unconstrained
Assume f has unique local minimum.
1 Find a_0, a_1 such that $a_0 \leq x^* \leq a_1$
f(0)
f(1)
f(2)
$f(x_n) > f(x_{n-1})$
0
1
2
x_{n-1}
$x_n = 2x_{n-1}$
$= a_1$
a_0



$a_2 = a_0 + \alpha^2(a_1 - a_0)$

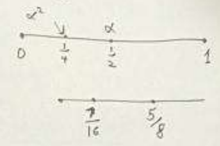
$a_3 = a_0 + \alpha(a_1 - a_0)$

Choose α

Interval shrinks by a factor $1-\alpha$ each iteration.

1 need one new function evaluation

$\alpha = \frac{1}{2}$



① $f(a_2) < f(a_3)$

$a_0' = a_0 \quad a_1' = a_3$

$a_2' = a_0' + \alpha^2(a_1' - a_0')$

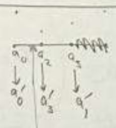
$a_3' = \underline{\underline{a_2}} = a_0' + \alpha(a_1' - a_0')$??

$\overset{?}{=}$

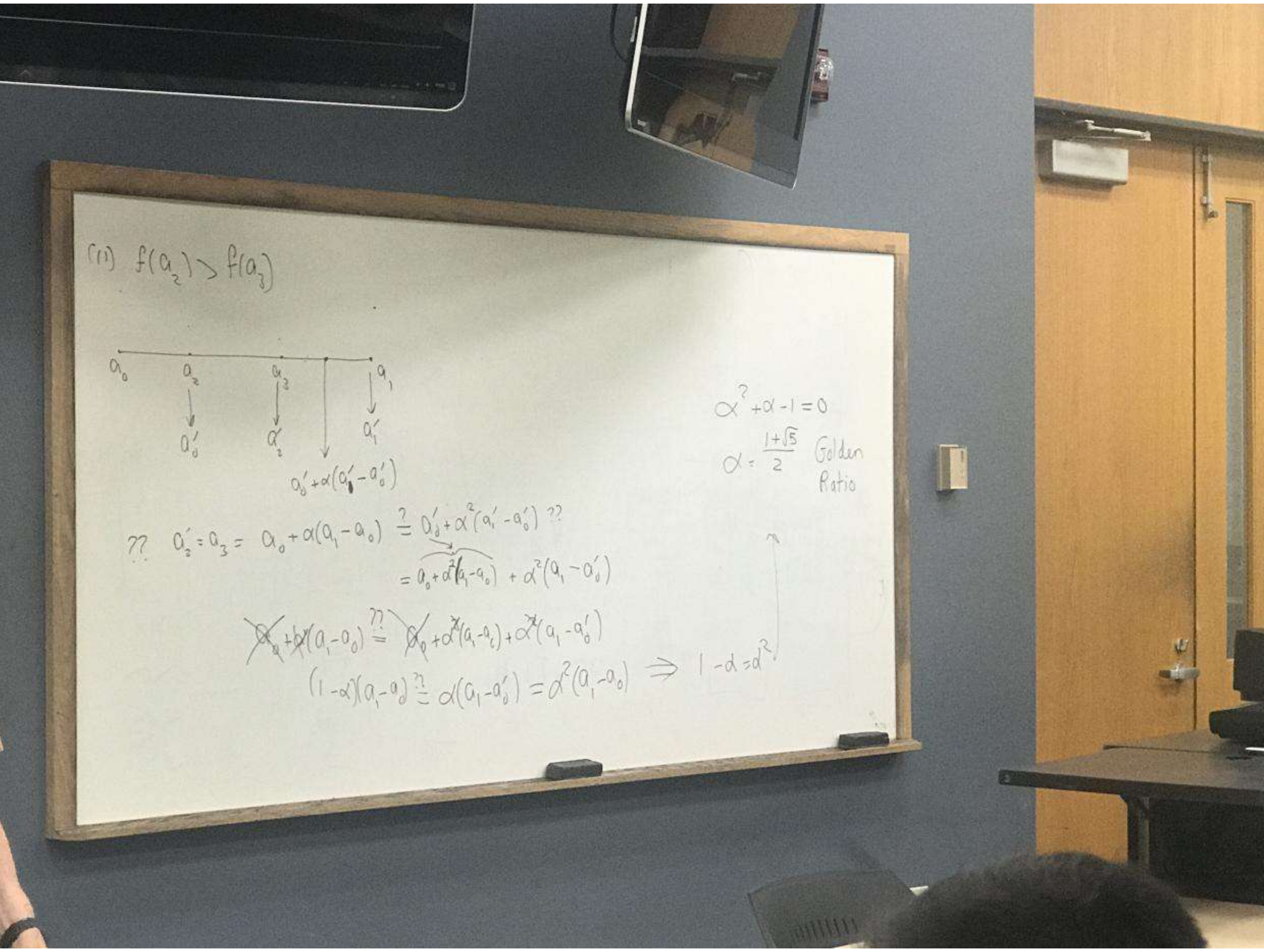
$a_2 = a_0 + \alpha^2(a_1 - a_0)$

$a_1' - a_0' = \alpha(a_1 - a_0)$

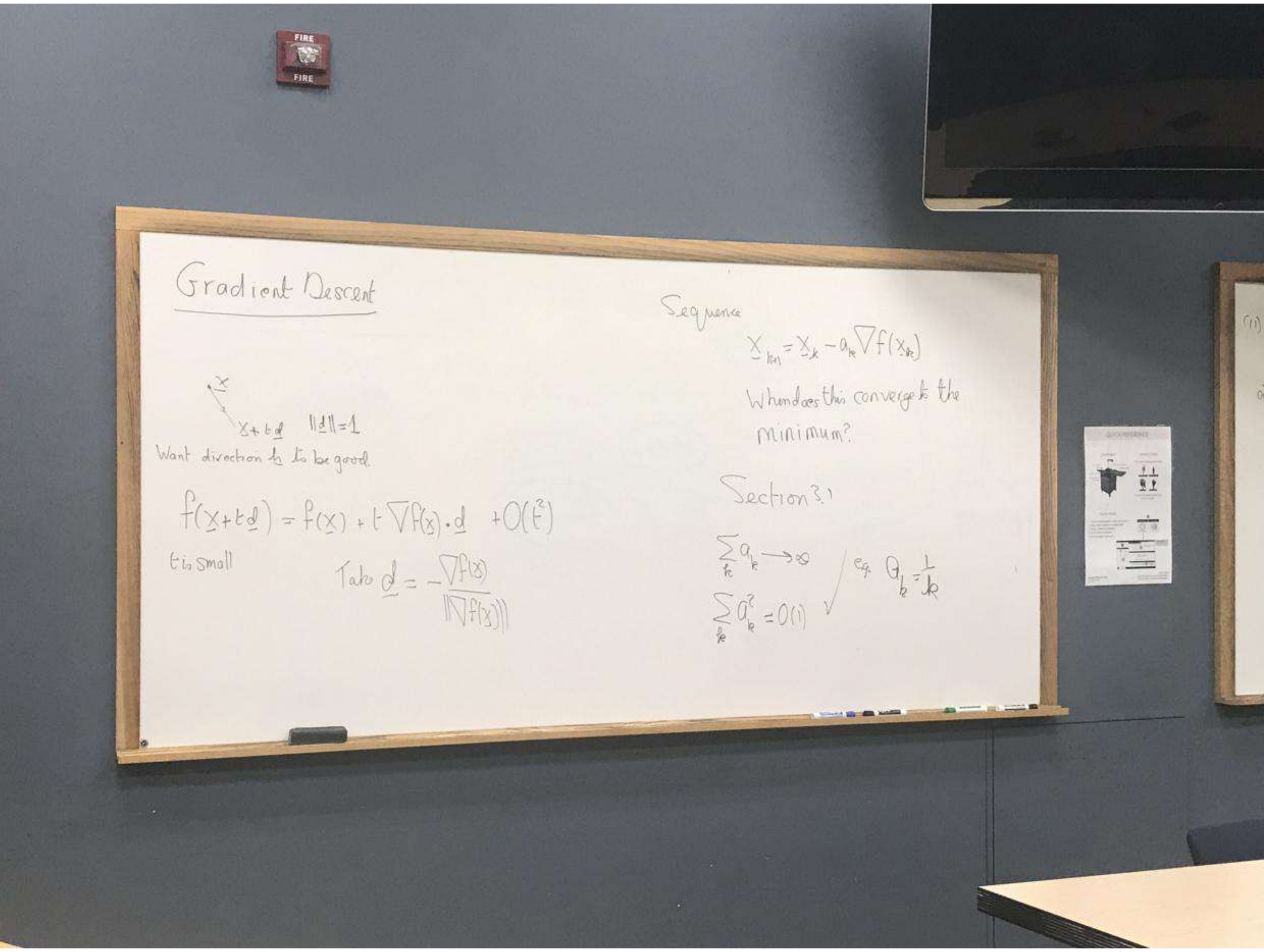
$\alpha(a_1 - a_1) = \checkmark$



One new function evaluation



(ii) $f(a_2) > f(a_3)$
a_0 a_2 a_3 a_1
a_0' a_2' a_1'
$a_0' + \alpha(a_1' - a_0')$
?? $a_2' = a_3 = a_0 + \alpha(a_1 - a_0) \stackrel{?}{=} a_0' + \alpha^2(a_1' - a_0')$??
$= a_0 + \alpha^2(a_1 - a_0) + \alpha^2(a_1 - a_0')$
$\cancel{a_0} + \cancel{\alpha}(a_1 - a_0) \stackrel{??}{=} \cancel{a_0} + \alpha^{\cancel{2}}(a_1 - a_0) + \alpha^{\cancel{2}}(a_1 - a_0')$
$(1-\alpha)(a_1 - a_0) \stackrel{??}{=} \alpha(a_1 - a_0') = \alpha^2(a_1 - a_0) \Rightarrow 1 - \alpha = \alpha^2$
$\alpha^2 + \alpha - 1 = 0$
$\alpha = \frac{1+\sqrt{5}}{2}$ Golden Ratio


FIRE
FIRE
Gradient Descent
x
$x+td$ $\|d\|=1$
Want direction to be good
$f(x+td) = f(x) + t\nabla f(x)\cdot d + O(t^2)$
t is small
Take $d = -\frac{\nabla f(x)}{\|\nabla f(x)\|}$
Sequence
$x_{k+1} = x_k - a_k \nabla f(x_k)$
When does this converge to the minimum?
Section 3.1
$\sum_k a_k \to \infty$
$\sum_k a_k^2 = O(1)$
e.g. $a_k = \frac{1}{k}$