Inventory Control

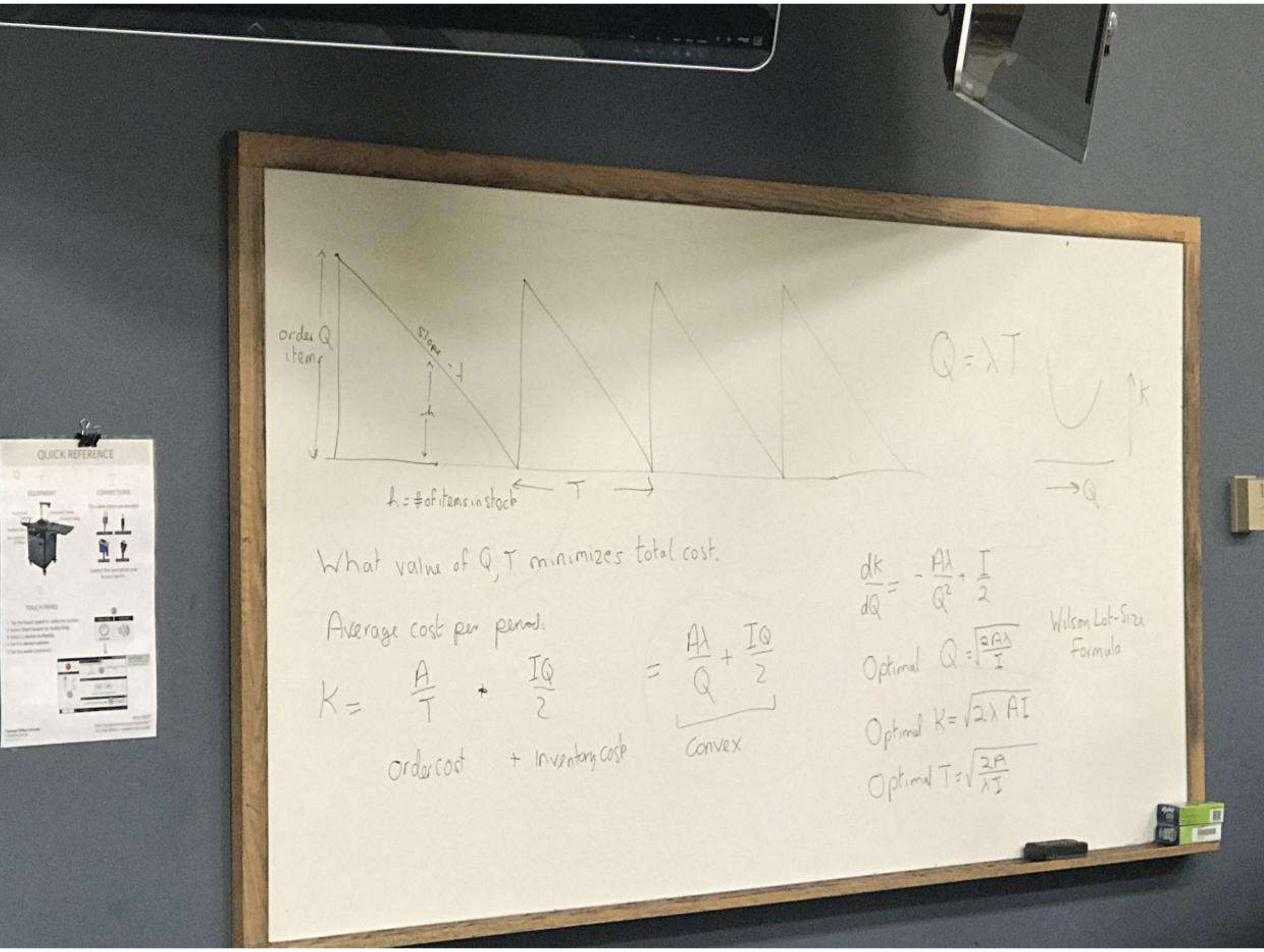
Model 1

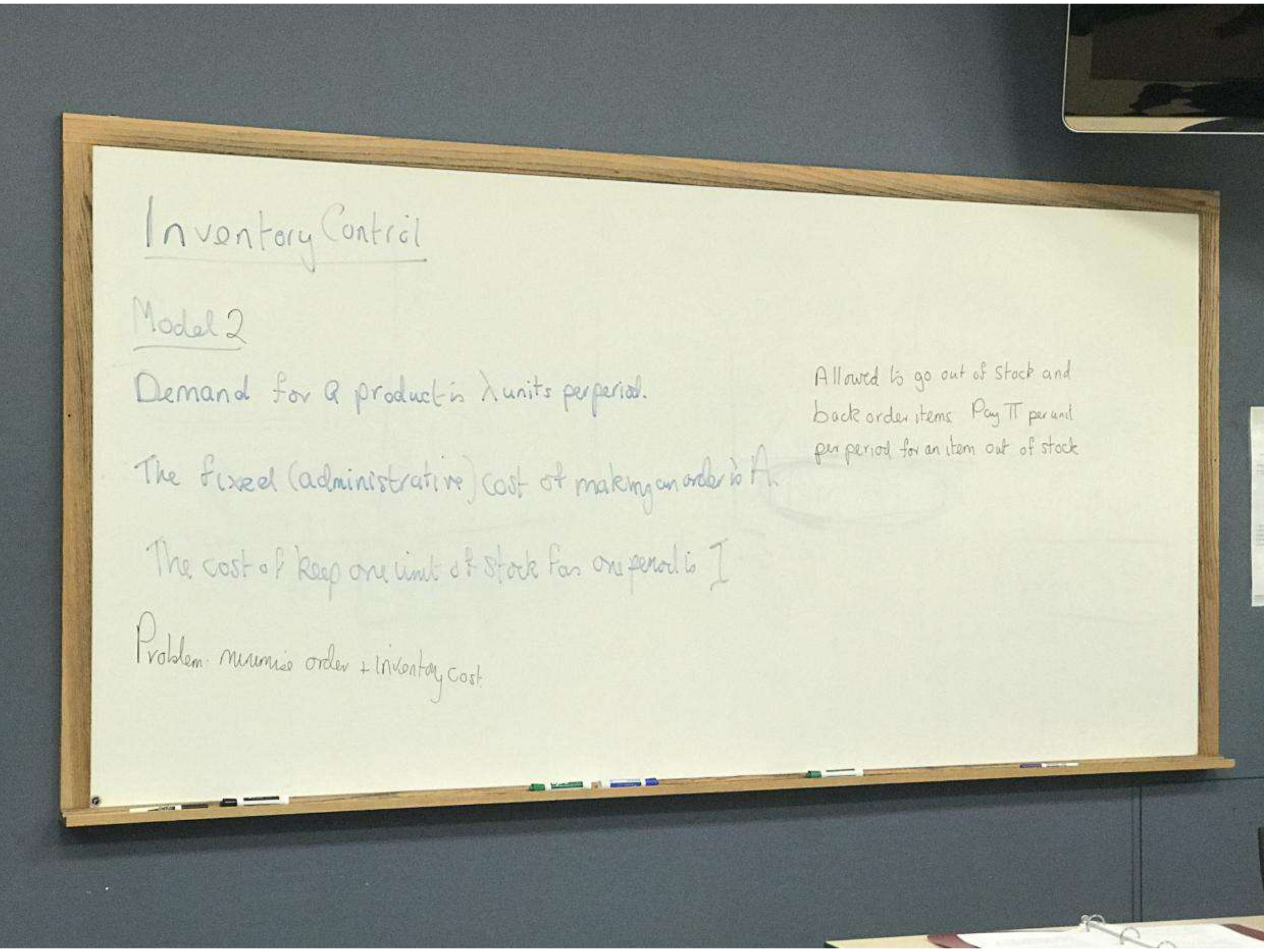
Demand for a product is λ units per period.

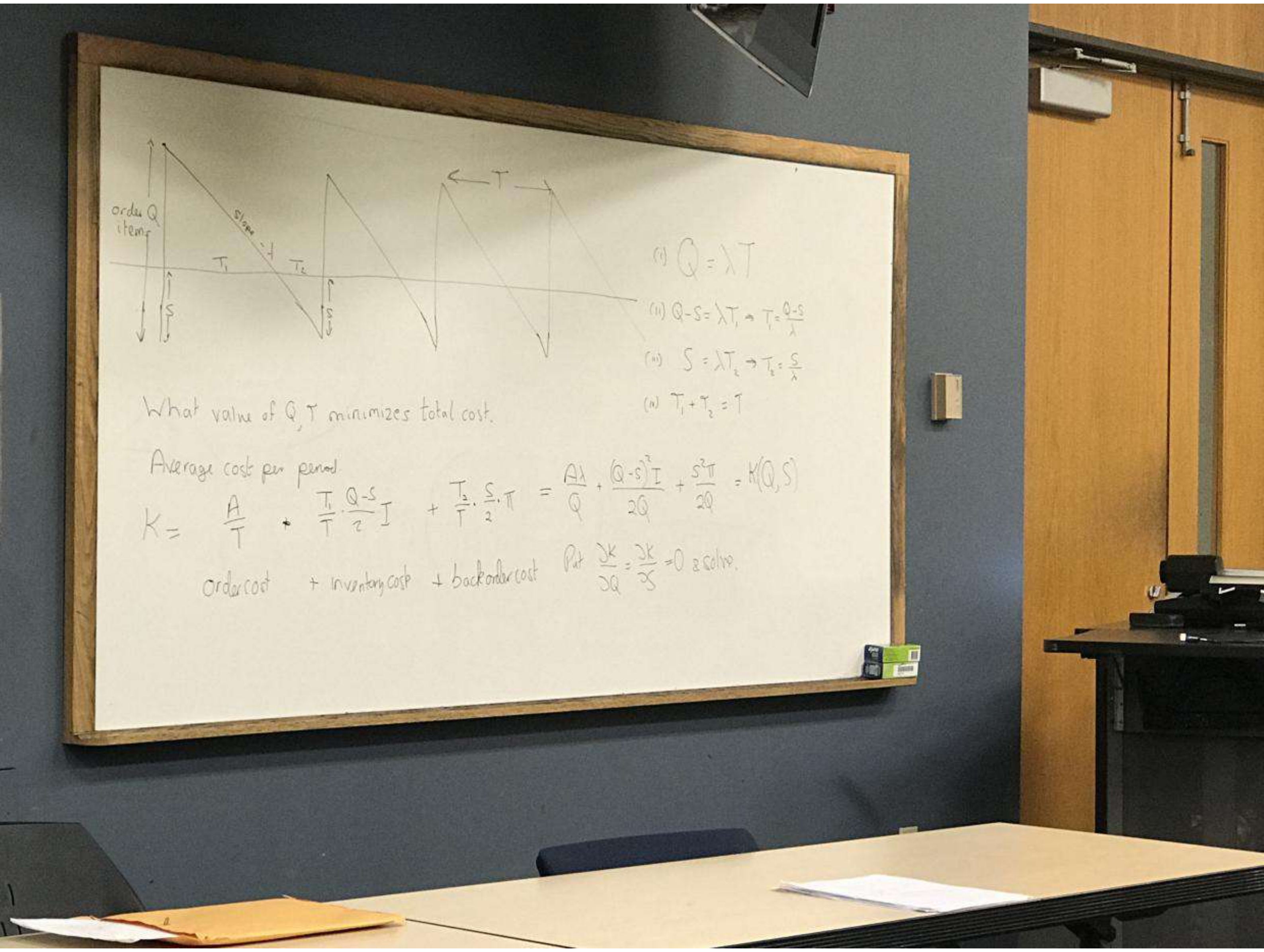
The fixed (administrative) cost of making an order is A.

The cost of keep one unit of stock for one period is I

Problem: minimise order + inventory cost.


order Q items
slope $-\lambda$
h
h = # of items in stock
$\leftarrow T \rightarrow$
$Q = \lambda T$
K
Q
What value of Q, T minimizes total cost.
Average cost per period:
$K = \frac{A}{T} + \frac{IQ}{2} = \underbrace{\frac{A\lambda}{Q} + \frac{IQ}{2}}_{\text{convex}}$
order cost + inventory cost
$\frac{dK}{dQ} = -\frac{A\lambda}{Q^2} + \frac{I}{2}$
Optimal $Q = \sqrt{\frac{2A\lambda}{I}}$
Wilson Lot-Size Formula
Optimal $K = \sqrt{2\lambda AI}$
Optimal $T = \sqrt{\frac{2A}{\lambda I}}$


Inventory Control
Model 2
Demand for a product is λ units per period.
The fixed (administrative) cost of making an order is A.
The cost of keep one unit of stock for one period is I
Problem: minimise order + inventory cost
Allowed to go out of stock and
back order items. Pay π per unit
per period for an item out of stock



order Q items
slope $-\lambda$
T_1
T_2
S
T
(i) $Q = \lambda T$
(ii) $Q - S = \lambda T_1 \Rightarrow T_1 = \frac{Q-S}{\lambda}$
(iii) $S = \lambda T_2 \Rightarrow T_2 = \frac{S}{\lambda}$
(iv) $T_1 + T_2 = T$
What value of Q, T minimizes total cost.
Average cost per period
$K = \frac{A}{T} + \frac{T_1}{T} \cdot \frac{Q-S}{2} I + \frac{T_2}{T} \cdot \frac{S}{2} \cdot \pi = \frac{A\lambda}{Q} + \frac{(Q-S)^2 I}{2Q} + \frac{S^2 \pi}{2Q} = K(Q, S)$
ordercost + inventory cost + backorder cost
Put $\frac{\partial K}{\partial Q} = \frac{\partial K}{\partial S} = 0$ & solve.

Inventory Control

Model 3

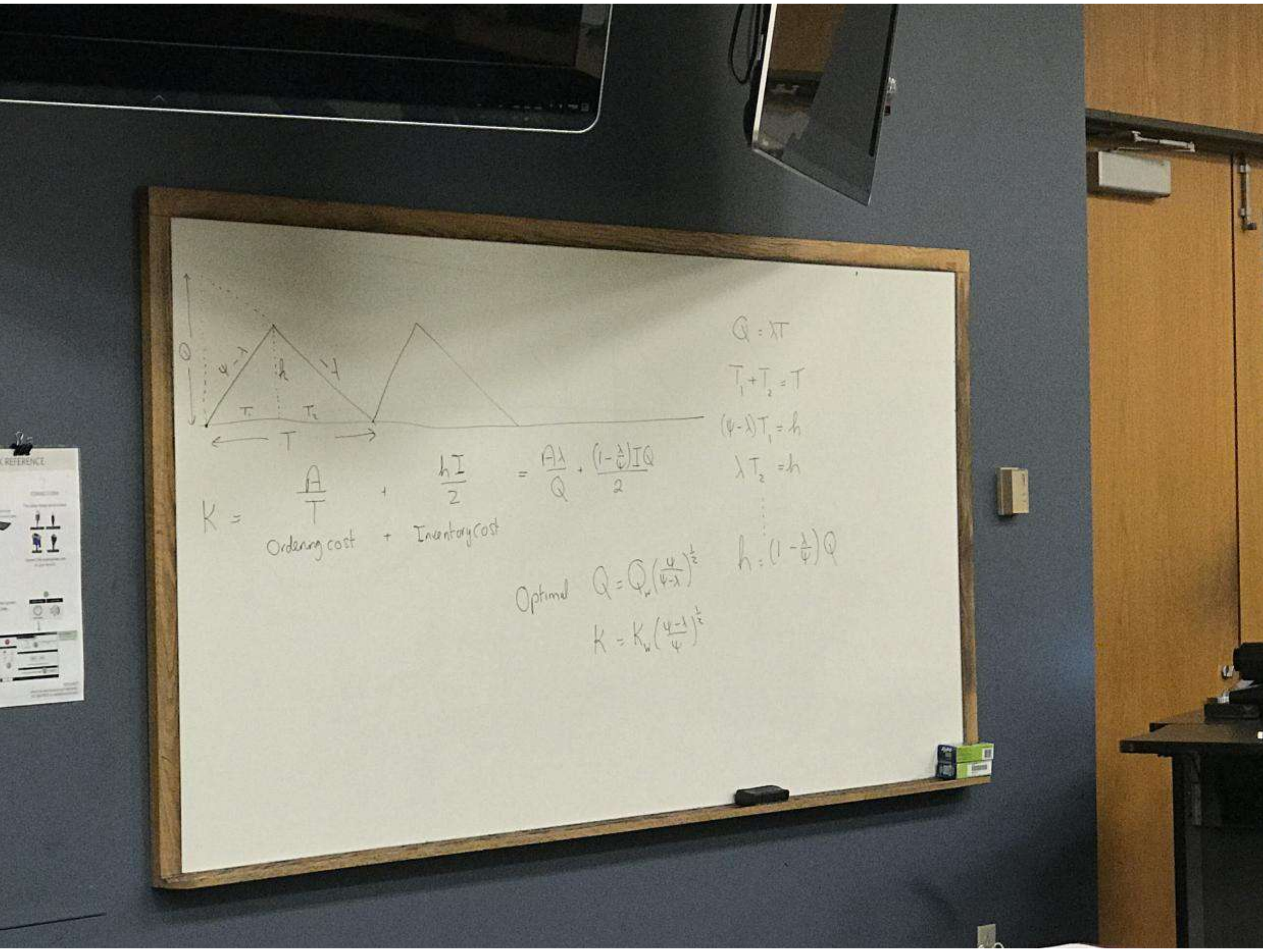
Items come in at a rate $\psi > \lambda$

Demand for a product is λ units per period.

The fixed (administrative) cost of making an order is A.

The cost of keep one unit of stock for one period is I

Problem: minimise order + inventory cost.



Q
$\psi-\lambda$
h
λ
T_1
T_2
T
$Q = \lambda T$
$T_1 + T_2 = T$
$(\psi-\lambda)T_1 = h$
$\lambda T_2 = h$
$K = \frac{A}{T} + \frac{hI}{2} = \frac{A\lambda}{Q} + \frac{(1-\frac{\lambda}{\psi})IQ}{2}$
Ordering cost + Inventory cost
$h = (1-\frac{\lambda}{\psi})Q$
Optimal $Q = Q_w\left(\frac{\psi}{\psi-\lambda}\right)^{\frac{1}{2}}$
$K = K_w\left(\frac{\psi-\lambda}{\psi}\right)^{\frac{1}{2}}$