Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 4: Due Monday October 27.

1. Consider the following in relation to the 0-1 Knapsack problem: For $i \in [n]$ let M_i be the value of the solution where (i) we fix $x_i = 1$ and (ii) we apply the greedy algorithm to find the remaining items. Let $M = \max\{M_1, M_2, \ldots, M_n\}$. Show that $M \geq Z_{OPT}/2$.

2. Give an algorithm to solve the following scheduling problem. There are n jobs labelled $1, 2, \ldots, n$ that have to be processed one at a time on a single machine. There is an acyclic digraph $D = (V, A)$ such that if $(i, j) \in A$ then job j cannot be started until job i has been completed. The problem is to minimise $\max_j f_j(C_j)$ where for all j, f_j is a monotone increasing. As usual, C_j is the completion time of job j. This is distinct from its processing time p_j.

3. Find the optimal ordering strategy for the following inventory system. If you order an amount Q, it costs AQ^α for some $0 < \alpha < 1$ and the inventory cost is I per unit per period. The demand is λ units per period and stock-outs are allowed. The penalty cost for stock-outs are π per unit per period.