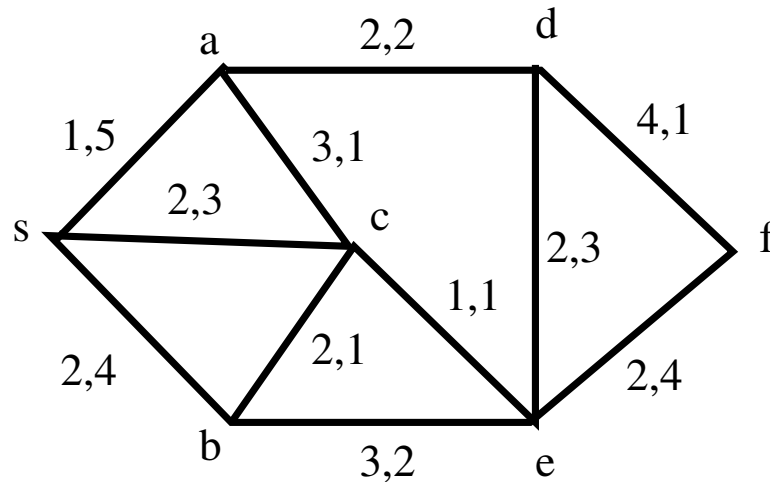


OPERATIONS RESEARCH II 21-393

Homework 2: Due Friday September 19.

- Find a shortest path from s to all other nodes in the digraph below. Each edge (x, y) is labelled by a pair (a, b) and the length of the corresponding arc is $a + bt$ where t is the time the path reaches x . All arcs are directed lexicographically e.g. (c, e) is directed from c to e .



- Let \mathcal{W} denote the set of walks in a directed graph D . If W_1 is a walk from a to b and W_2 is a walk from b to c then $W_1 + W_2$ is the walk from a to c obtained by following W_1 and then W_2 .

Let $\ell : \mathcal{W} \rightarrow \mathbb{R}$ be a real valued function defined on \mathcal{W} . Suppose that it has the following properties:

- $\ell(C) \geq 0$ for any closed walk C . (A walk is closed if it begins and ends at the same vertex).
- If W_1, W'_1 are walks from a to b and W_2, W'_2 are walks from b to c and $\ell(W'_i) \geq \ell(W_i)$ for $i = 1, 2$ then $\ell(W'_1 + W'_2) \geq \ell(W_1 + W_2)$.

Consider the following algorithm: n is the number of vertices in D .

Initialise $W_{i,j} = (i, j)$ and $D_{i,j} = \ell(W_{i,j})$ for $i, j = 1, 2, \dots, n$.

```

For  $k = 1$  to  $n$  Do
  For  $i = 1$  to  $n$  Do
    For  $j = 1$  to  $n$  Do
       $D_{i,j} \leftarrow \min\{D_{i,j}, \ell(W_{i,k} + W_{k,j})\}$ 
      If  $D_{i,j} = \ell(W_{i,k} + W_{k,j})$  then  $W_{i,j} \leftarrow W_{i,k} + W_{k,j}$ 
    oD
  oD
oD

```

Prove that when the algorithm finishes,

$$D_{i,j} = \min\{\ell(P) : P \text{ is a path from } i \text{ to } j\}.$$

3. Find a minimum weight spanning tree in the weighted graph below:

