Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 1: Due Monday September 15.

Q1 Solve the following knapsack problem:

$$\begin{array}{lrcrcrcl} \text{maximise} & 4x_1 & + & 8x_2 & + & 13x_3 & & \\ \text{subject to} & & & & & & & \\ & 3x_1 & + & 4x_2 & + & 5x_3 & \leq & 16 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

Q2 Consider a 2-D map with a horizontal river passing through its center. There are n cities on the southern bank with x-coordinates $a(1)...a(n)$ and n cities on the northern bank with x-coordinates $b(1)...b(n)$. You want to connect as many north-south pairs of cities as possible with bridges such that no two bridges cross. When connecting cities, you can only connect city i on the northern bank to city i on the southern bank. Construct a Dynamic Programming solution to this problem. (You can assume that $a(1) < a(2) < \cdots < a(n)$, but you **cannot** assume that $b(1) < b(2) < \cdots < b(n)$. If both sequences are increasing, then the problem is trivial).

Q3 The people of a certain area live at the side of a long straight road of length L. The population is clustered into several villages at points $a_1, a_2, \ldots, a_k$ along the road. There is a proposal to build ℓ fire stations on the road. The problem is build them so that the maximum distance of a village to its nearest fire station is minimised. Formulate the problem of finding the optimum placement of fire stations as a dynamic program. (Assume that fire stations are to be placed at integer points only on the line.)