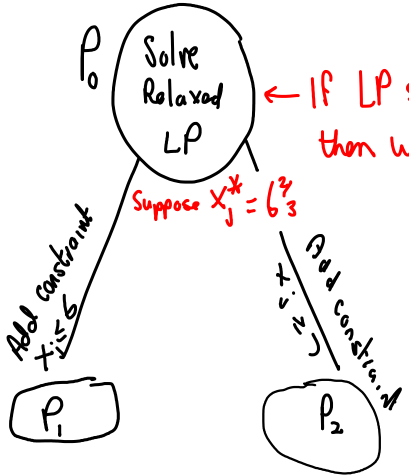
10\6\14

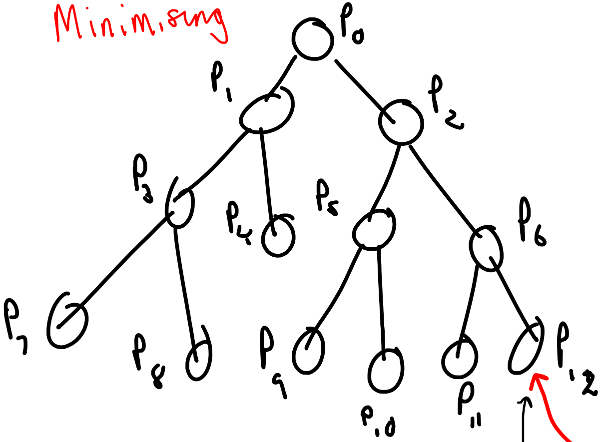
BRANCH & BOUND



← If LP solution x^* is integral then we stop

Solve P_1 & P_2

Minimising



Solve LP associated with this problem

(i) LP is infeasible.

- P_{12} is solved

(ii) LP comes out integer

- P_{12} is solved

(iii) We know some integer solution of cost c^* & LP value $c \geq c^*$

- we can drop P_{12}

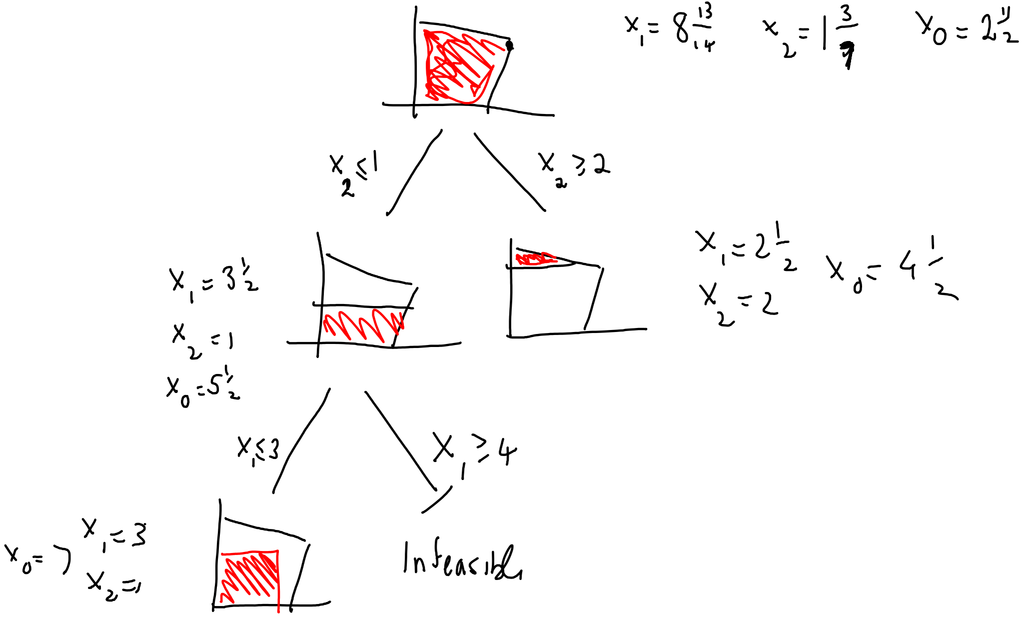
(iv) Otherwise we branch

Minimise $20 - 3x_1 - 4x_2$

s.t. $\frac{2}{5}x_1 + x_2 \leq 3$

$\frac{2}{5}x_1 - \frac{2}{5}x_2 \leq 1$

$x_1, x_2 \geq 0$ & integer



$x_1 = 8\frac{13}{14}$ $x_2 = 1\frac{3}{7}$ $x_0 = 2\frac{1}{2}$
$x_2 \leq 1$
$x_2 \geq 2$
$x_1 = 3\frac{1}{2}$
$x_2 = 1$
$x_0 = 5\frac{1}{2}$
$x_1 = 2\frac{1}{2}$
$x_2 = 2$
$x_0 = 4\frac{1}{2}$
$x_1 \leq 3$
$x_1 \geq 4$
$x_0 = 7$ $x_1 = 3$
$x_2 = 1$
Infeasible

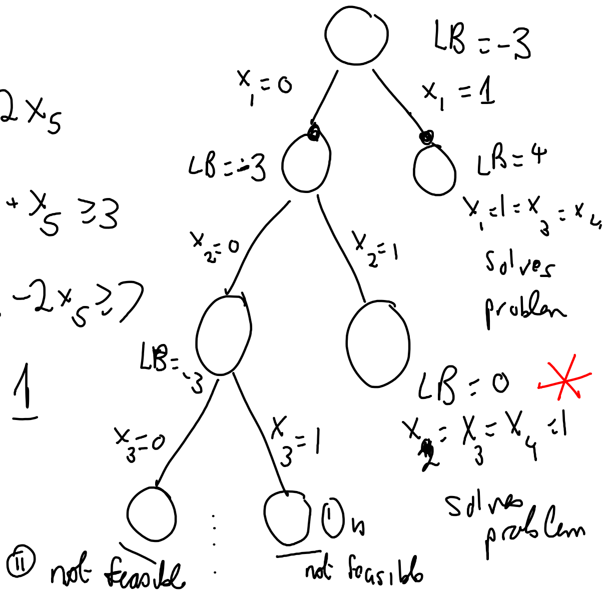
Implicit Enumeration

Minimise $Z = 7x_1 + 3x_2 + 2x_3 - x_4 - 2x_5$

(I) $4x_1 + 12x_2 - 10x_3 + 2x_4 + x_5 \geq 3$

(II) $12x_1 + 12x_2 + 4x_3 - x_4 - 2x_5 \geq 7$

$x_1, \ldots, x_5 = 0$ or $\underline{1}$



Approximation Algorithms

0-1 Knapsack Problem: maximize $p_1x_1 + \dots + p_nx_n$

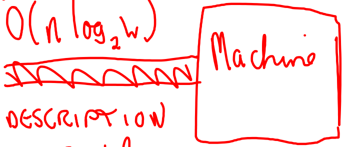
s.t. $w_1x_1 + \dots + w_nx_n \le W$

$x_i = 0$ or 1

Can solve this in $O(nW)$ time.

Assume $p_1 = \dots = p_n = 1$

$O(n \log_2 W)$



DESCRIPTION of Problem

Algorithm takes

Not poly time

0-1 Knapsack is NP-hard

Heuristics: Describe heuristic, which runs in poly time.
See how well it does.

Heuristic 1: Assume $\frac{p_1}{w_1} \geq \frac{p_2}{w_2} \geq \dots \geq \frac{p_n}{w_n}$

Try adding variables in order 1, 2, . . .

$$\max \quad 12x_1 + 9x_2 + 7x_3 + 5x_4 + 3x_5$$

$$x_1 + 2x_2 + 3x_3 + 4x_4 + x_5 \leq 8$$

Go through left to right trying to make $x_j = 1$: $x_1 = 1, x_2 = 1, x_3 = 1,$
$x_4 = 0, x_5 = 1$