10\24\14

A can always win P_A

B can always win P_B

There is a stable solution iff $P_A = P_B$

(Nash Equilibrium)

Given matrix A. $(m \times n)$.

A: mixed strategy $p = (p_1, p_2, \cdots, p_m)$ for A $\quad p_1 + \cdots + p_m = 1$

$p_i \geq 0$

involves: A plays i with probability p_i.

B: mixed strategy $q = (q_1, q_2, \cdots q_n)$

B plays j with probability q_j

$$\begin{array}{cc} & \underline{q} \\ \underline{p} & \begin{bmatrix} \\ PAY(p,q) \\ \\ \end{bmatrix} \end{array}$$

expected payoff

$$PAY(p,q) = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}\, p_i\, q_j$$

$$P_A = \max_{\underline{p}} \min_{\underline{q}} \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \, p_i \, q_j$$

$$= \max_{\underline{p}} \left(\min_{\underline{q}} \sum_{j=1}^{n} c_j(p) \, q_j \right) \quad \text{where } c_j(p) = \sum_{i=1}^{m} a_{ij} \, p_i$$

$$= \max_{\underline{p}} \min_{j=1,\dots,n} c_j(p)$$

$$\min_{p} \; 3p_1 + 4p_2 + 2p_3$$

$$= 2$$

$$p_A = \max_{\underline{p}} \min \{ c_1(p), c_2(p), \dots, c_n(p) \}$$

$$= \max \xi$$

$$\xi \leq c_1(p) = \sum_{j=1}^{n} a_{i\underline{1}} p_i$$

$$\vdots$$

$$\xi \leq c_n(p) = \sum_{j=1}^{m} a_{in} p_i$$

$$p_1 + \dots + p_m = \underline{1} \qquad p_1, \dots, p_m \geq 0$$

$$P_B = \min_{q} \max_{p} PAY(p, q)$$

$$= \min \eta$$

$$\eta \geq \sum_{j=1}^{n} a_{1j} q_j$$

$$\vdots$$

$$\eta \geq \sum_{j=1}^{n} a_{mj} q_j$$

$$q_1 + \cdots + q_n = 1 \qquad q_1, \ldots, q_n \geq 0$$

$$\max \xi$$

$$\xi \leq c_1(p) = \sum_{j=1}^{n} a_{11} p_i$$

$$\vdots$$

$$\xi \leq c_n(p) = \sum_{j=1}^{m} a_{in} p_i$$

$$p_1 + \cdots + p_m = 1 \qquad p_1, \ldots, p_m \geq 0$$

$$\min \eta$$

$$\eta \geq \sum_{j=1}^{n} a_{1j} q_j$$

$$\vdots$$

$$\eta \geq \sum_{j=1}^{n} a_{mj} q_j$$

$$q_1 + \cdots + q_n = 1 \qquad q_1, \ldots, q_n \geq 0$$

These LP's are dual and so $\rho_A = \rho_B$.