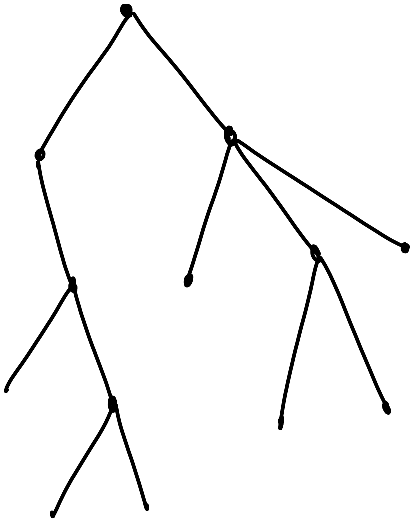
9\10\14

Suppose now that we must cut stick into exactly k pieces

$$f_r(\ell) = \max_{x<\ell} \left[v_{x,\ell} + f_{r-1}(x) \right]$$

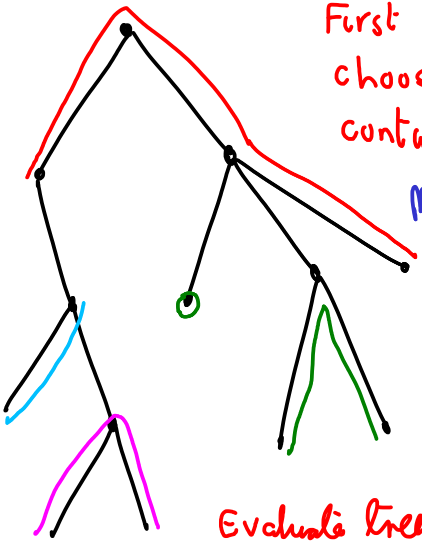
$1 \le r \le k$ $\quad$ $0 \le \ell \le L$

place to cut



Each path in tree has a value

Break up tree into vertex disjoint paths to maximise total value.

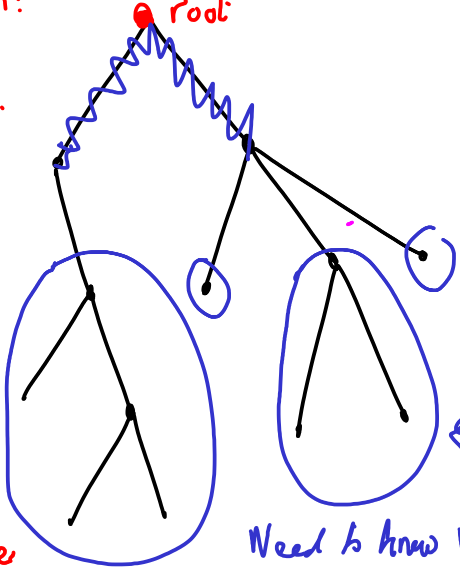


First decision: choose path containing root.

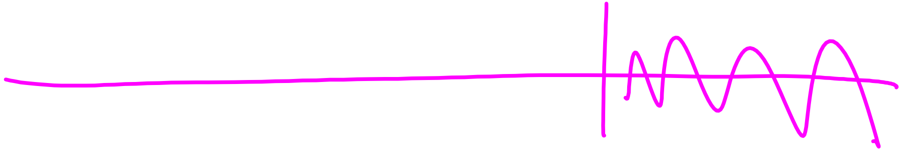
Maximize over choice of path

Evaluate trees with lowest root first order

root



Need to know value



$$f(l) = \max_{x} \left[v_{x,l} + f(x) \right]$$

Combinatorial Optimization:

(i) Minimum length spanning tree

(ii) Shortest path

(iii) Assignment problem.

Minimum Spanning Tree

Given a connected graph G with edge weights

Problem: find spanning tree of minimum total weight

Kruskal's Algorithm:

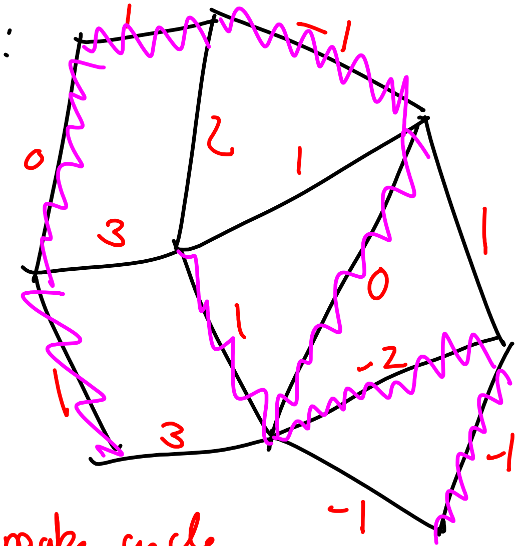
Greedy Algorithm:

Take edge of G in order

$e_1, e_2, \ldots$

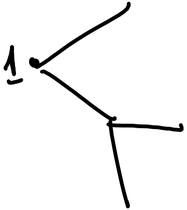
where $w(e_i) \leq w(e_{i+1})$

Include edge if does not make cycle.

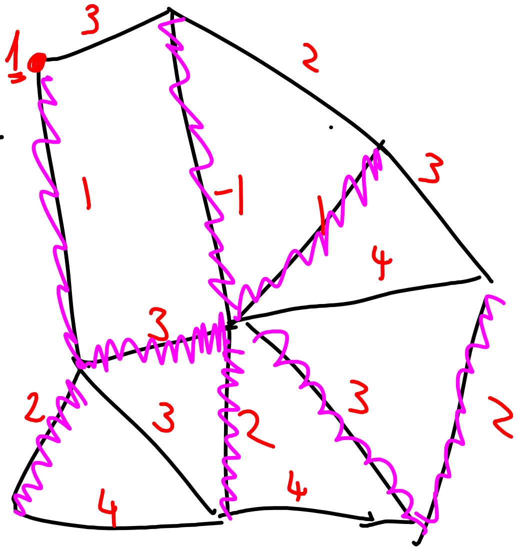


Dijkstra's Algorithm

Having constructed partial tree T



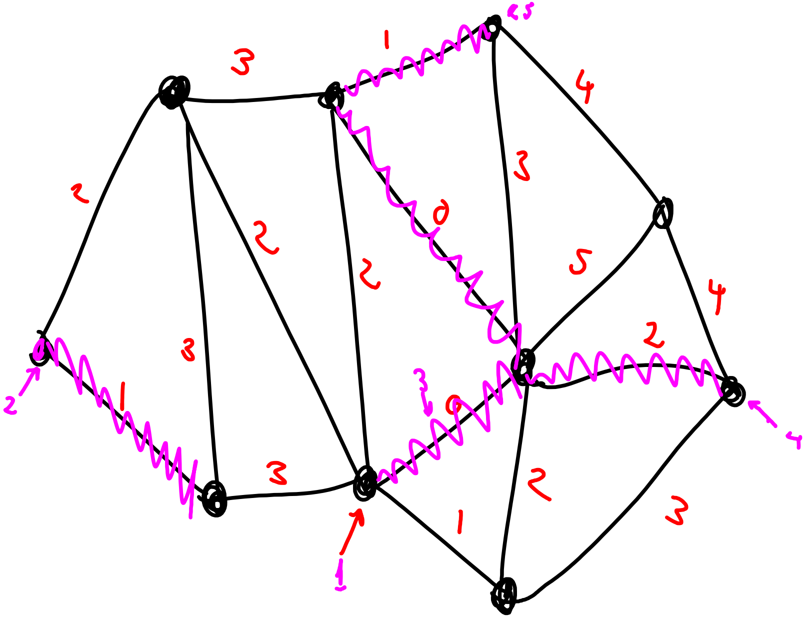
Added shortest edge from T to $\overline{T}$



More general algorithm: Having chosen

$F_t = \{e_1, e_2, \dots, e_t\}$ = Collection of disjoint subtrees $T_1, T_2, \dots, T_k$

Choose any T_i and add the shortest edge from T_i to $\overline{T_i}$.



3
1
2.5
4
2
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3
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2
3
5
4
3
2
2
1
3
0
4
3
2
1
1
3