9/8/14

Maximize $x_1 + 5x_2 + 9x_3 + 15x_4$

$$x_1 + 3x_2 + 4x_3 + 6x_4 \leq 9$$

$x_1, \ldots, x_4 \geq 0$ & integer

w	f_1	b_1	f_2	b_2	f_3	b_3	f_4	b_4
0	0	0	0	0	0	0	0	0
1	1	1	1	0	1	0	1	0
2	2	1	2	0	2	0	2	0
3	3	1	3 ← b=0 *5 ← b=1	1	5	0	5	0
4	4	1	6	1	9	1	9	0
5	5	1	7	1	10	1	10	0
6	6	1	10	1	11	1	15	1
7	7	1	11	1	14	1	16	1
8	8	1	12	1	18	1	18	0
9	9	1	15	1	19	1	20	1

$x_1 = 0$

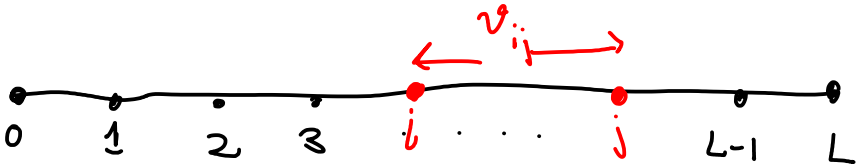
$x_2 = 1$

$x_3 = 0$

$x_4 = 1$

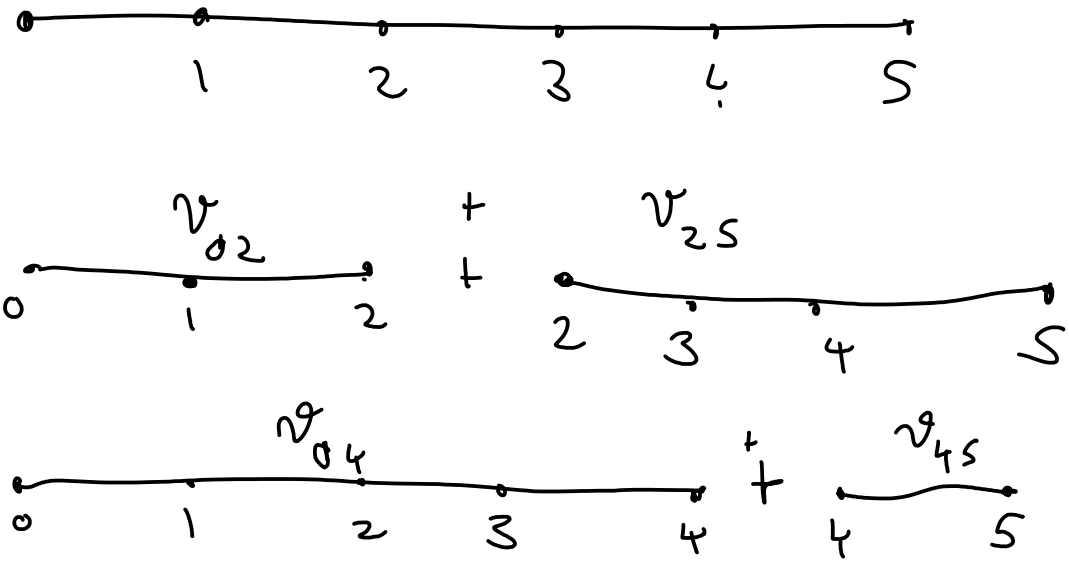
$b_i = 0$ mean no more x_i

Breaking up a stick



For each $i < j$ there is a value v_{ij} of the sub-stick

Problem: break up stick to maximize total value of all the pieces


1
2
3
4
5
v_{02}
+
v_{25}
0
1
2
2
3
4
5
v_{04}
+
v_{45}
0
1
2
3
4
4
5

$f(l)$ = max. value from cutting stick

$0 \leftrightarrow l$

$0 \leq l \leq L$

$f(0) = 0$

$$= \max_{0 \leq x \leq l-1} \left[v_{x,l} + f(x) \right]$$

- Execution time $= O(L^2)$

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & 2 & 5 & 1 \\ & 0 & 2 & 4 & 3 \\ & & 0 & 5 & 2 \\ & & & 0 & 4 \\ & & & & 0 \end{bmatrix} \end{array}$$

$L = 4$

$f(1) = 3$

$f(2) = \max\{\underset{x=1}{2+f(1)}, \underset{x=0}{2+f(0)}\}$

$= 5$

$f(3) = \max\{\underset{x=2}{5+f(2)}^{*}, \underset{x=1}{4+f(1)}, \underset{x=0}{5+f(0)}\}$

$= 10$

$f(4) = \max\{\underset{x=3}{4+f(3)}^{*}, \underset{x=2}{2+f(2)}, \underset{x=1}{3+f(1)}, \underset{x=0}{1+f(0)}\} = 14$

Suppose now that we must cut stick into exactly k pieces