9/05/14

Knapsack Problem:

maximise $c_1 x_1 + \cdots + c_n x_n$

s.t. $w_1 x_1 + \cdots + w_n x_n \leq h$

$x_1, \ldots, x_n \geq 0$ & integer

$f_i(w) =$ max. $c_1 x_1 + \cdots + c_r x_r$

$w_1 x_1 + \cdots + w_r x_r \leq w$

$x_1, \ldots, x_r \geq 0$ & integer

$$f_i(w) = \max_{0 \le x_r \le \left\lfloor \frac{w}{w_r} \right\rfloor} \left[c_i x_i + f_{r-1}(w - w_r x_r) \right]$$

$$f_1(w) = c_1 \left\lfloor \frac{w}{w_1} \right\rfloor$$

or

$f_0(w) = 0$

Execution time =

$O(nW^2)$

Alternative recurrence:

$$f_r(w) = \max \begin{cases} x_r = 0 & f_{r-1}(w) \quad b=0 \\ x_r \geq 1 & c_r + f_r(w - w_r) \quad b=1 \end{cases}$$

Execution time $O(nW)$

Maximize $x_1 + 5x_2 + 9x_3 + 15x_4$

$$x_1 + 3x_2 + 4x_3 + 6x_4 \leq 9$$

$$x_1, \ldots, x_4 \geq 0 \text{ \& integer}$$

w	f_1	b_1	f_2	b_2	f_3	b_3	f_4	b_4
0	0	0	0	0	0	0		
1	1	1	1	0	1	0		
2	2	1	2	0	2	0		
3	3	1	3 ← b=0, 5 ← b=1	1	5	0		
4	4	1	6	1	9	1		
5	5	1	7	1	10	1		
6	6	1	10	1	11	1		
7	7	1	11	1	14	1		
8	8	1	12	1	18	1		
9	9	1	15	1				

$b_i = 0$ mean no more x_i