9\3\14

Example of production problem.

$H = 3$, $n = 5$, $c(x) = 18x - x^2$.

$d_i = 4$, $\forall i$

$$f_r(i) = \min_x \left[c(x) + f_{r+1}(i + x - d_r) \right]$$

i	f_1	x_1	f_2	x_2	f_3	x_3	f_4	x_4	f_5	x_5
0	244	4	188	7	150	4	112 110 104 *94	4 5 6 7	56	4
1	233	3	183	6	139	3	101 101 97 *89	3 4 5 6	45	3
2	220	2	176	5	126	2	94	5	32	2
3	205	1	167	1	111	1	89	1	17	1

Variations

① Add a smoothing cost $\sigma(x_1, x_2)$ for making x_1, x_2 in succesive periods

e.g. $\sigma(x_1, x_2) = k(x_1 - x_2)^2$

$$f_r(i, y) = \min_{x} \left[c(x) + \sigma(y, x) + f_{r+1}(i + x - d_r, x) \right]$$

prod. last period

iii) Machine replacement.

Suppose cost of producing x depends on the age of the machine : $C(x,t)$ for a machine age t.

Cost of new machine is A.

Must acquire new machine once machine reaches T, otherwise it is optional.

$$f_r(i,t) = \min \begin{cases} \text{keep} & \min_{x}\left(c(x,t) + f_{r+1}(i+x-d_r, t+1) \right) \\ \text{replace} & A + \min_{x}\left(c(x,0) + f_{r+1}(i+x-d_r, 1) \right) \end{cases}$$

age of machine

$$f_r(i,T) =$$

Knapsack problem

Problem: choose items to maximize value

Scout X is going to camp.

Has a knapsack.

X can carry at most weight W

n possible items to pack. Each item of type j has weight w_j and value c_j.

Integer program: Maximize $c_1x_1 + c_2x_2 + \cdots + c_nx_n$

x_i := # items of type i that X takes with them.

$$w_1x_1 + w_2x_2 + \cdots + w_nx_n \leq W$$

$$x_i \in \{0, 1, 2, \ldots\}$$

Dynamic Programming: Sequence of decisions $x_1, x_2, \ldots$

$$\text{Opt}[1,2,3,\ldots,n; W] = \max_{x_1} \left[c_1x_1 + \text{Opt}[2,3,\ldots,n; W - w_1x_1 \right.$$

$f_r(W)$ = max. obtainable using items of type $1, 2, \ldots, r$ and knapsack of size W

$$= \max_{x_r} \left[c_r x_r + f_{r-1}(W - w_r x_r) \right]$$