## Department of Mathematical Sciences

CARNEGIE MELLON UNIVERSITY

## OPERATIONS RESEARCH II 21-393

Homework 2: Due Wednesday September 25.

1. Formulate the following as an integer program: There are $n$ students and exams $E_{1}, E_{2}, \ldots, E_{m} \subseteq[n]$ need to be scheduled. Here $E_{i}$ is the set of students that need to take exam $i$. Exams take place in the morning and in the afternoon. There are $s$ rooms available and each room can hold $r$ students. The rules are (i) A student must not be asked to take more than one exam per day; (ii) Several different exams can be held in the same room at the same time provided there is capacity in the room to hold the students. The problem is to minimise the number of days needed to carry out all of the exams.
2. Solve the following problem by a cutting plane algorithm:

$$
\begin{aligned}
& \operatorname{minimise} 4 x_{1} \\
& \text { subject to } \\
& 2 x_{1}+3 x_{3} \\
&+x_{2} \quad-x_{3} \geq 2 \\
& x_{1}+4 x_{2}+x_{3} \geq 13 \\
& x_{1}, x_{2}, x_{3} \geq 0 \text { and integer. }
\end{aligned}
$$

3. Solve the following problem by a branch and bound algorithm:

$$
\text { Maximise } \quad 4 x_{1} \quad-2 x_{2} \quad+7 x_{3} \quad-x_{4}
$$

subject to

$$
\begin{array}{lll}
x_{1} & +5 x_{3} & \leq 10 \\
x_{1} & +x_{2} & -x_{3} \\
6 x_{1} & -5 x_{2} & \leq 1 \\
-x_{1} & & \leq 2 x_{3} \\
& -2 x_{4} & \leq 3 \\
& \\
x_{1}, x_{2}, x_{3}, x_{4} \geq 0 . & \\
x_{1}, x_{2}, x_{3} \text { integer. } &
\end{array}
$$

