Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 1: Due Wednesday September 11.

Q1 Solve the following knapsack problem:

$$\begin{array}{lccccc} \text{maximise} & 4x_1 & + & 8x_2 & + & 15x_3 & & \\ \text{subject to} & & & & & & & \\ & 3x_1 & + & 4x_2 & + & 5x_3 & \leq & 19 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

Q2 An $m \times n$ rectangle of wood is to be cut into smaller rectangles. An $a \times b$ rectangle is worth $m_{a,b}$. The machine that cuts rectangles can only cut full length or full width. I.e. if after cutting there is an $x \times y$ rectangle then the machine can cut it into two rectangles $z \times y$ and $(x - z) \times y$ for some z or into two rectangles $x \times z$ and $x \times y - z$.
Describe a dynamic programming algorithm for finding the way of cutting into pieces that maximises the total value of the rectangles produced.
Q3 We are given $2n$ sets $D_1, D_2, \ldots D_n$ and $R_1, R_2, \ldots, R_n$ where n is even. Also, $|D_i| + |R_i| = m$ for $i = 1, 2, \ldots, n$. Find an algorithm that will check to see if the following is possible: Find a set $I \subseteq [n]$, $|I| = n/2$ such that

$$\sum_{i \in I} |D_i| \geq \sum_{i \in I} |R_i| \text{ and } \sum_{i \notin I} |D_i| \geq \sum_{i \notin I} |R_i|.$$

Your algorithm should run in time polynomial in m, n.
Hint: For $I \subseteq [n]$ let $D_I = \sum_{i \in I} |D_i|$ and $R_I = \sum_{i \in I} |R_i|$. Then let

$$f_{k,\ell}(a, b, c, d) = \begin{cases} 1 & \exists I \subseteq [k] : |I| = \ell \text{ and } D_I = a, D_{[k]\setminus I} = b, R_I = c, R_{[k]\setminus I} = d \\ 0 & \textit{otherwise} \end{cases}$$

Solve the problem by establishing a recurrence for f.