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Integer Linear Programming

Minimise $\underline{c}^T \underline{x}$

subject $A\underline{x} = \underline{b}$

A is an m×n matrix

$\underline{x} \geq 0$

x_j is integer for $j \in I \subseteq \{1, 2, \dots, n\}$

(1) Capital Budgeting

Planning next m periods: b_i dollars avaible for period i

n possible projects.

If we undertake projection j, then it will produce profit p_j.

In period i, it requires a_{ij} dollars.

Problem: which projects should be undertaken to max. profit.

For every j we have to decide whether or not to go ahead with project j.

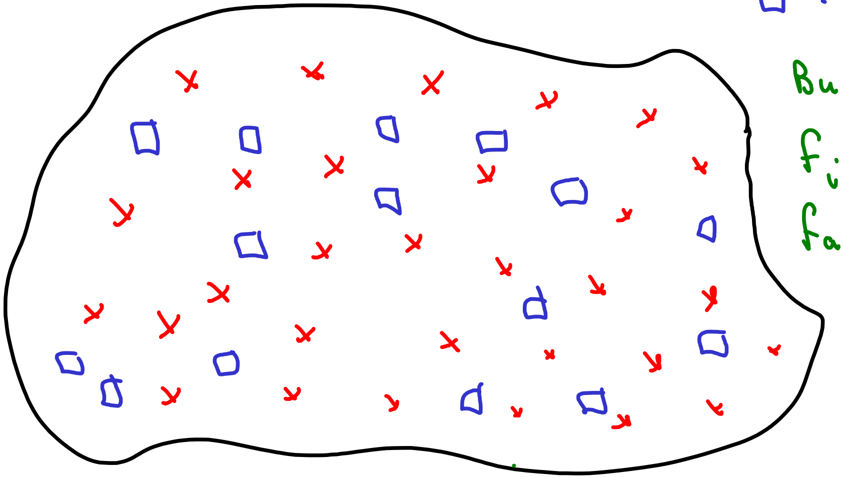
Variable $x_j = \begin{cases} 0 & : \text{do not do project } j \\ 1 & : \text{do project } j \end{cases}$

Maximize profit $= p_1 x_1 + p_2 x_2 + \cdots + p_n x_n$

$$a_{i,1} x_1 + a_{i,2} x_2 + \cdots + a_{i,n} x_n \leq b_i \quad , i = 1, 2, \ldots m$$

$x_j = 0$ or 1 $\qquad 0 \leq x_j \leq 1, \quad x_j$ integer

Plant Location problem



× ≡ customer

□ : possible site for a plant.

Building cost + transhipment cost

f_i to make facility i

c_{ij} = cost of supply j from i.

$$y_i = \begin{cases} 0 : \neg\text{build} \\ 1 : \text{build} \end{cases}$$

x_{ij} = proportion of j supplied by i.

minimise $\underbrace{f_1 y_1 + f_2 y_2 + \cdots + f_m y_m}_{\text{building cost}} + \underbrace{\sum_i \sum_j c_{ij} x_{ij}}_{\text{transport cost}}$

Subject to $\displaystyle\sum_{i=1}^{m} x_{ij} = 1$ for all i

$x_{ij} > 0 \Rightarrow y_i = 1$ $\quad 0 \leq x_{ij} \leq y_i$

$y_i = 0$ or 1

Set Cover Problem

Sets $S_1, S_2, \ldots, S_n \subseteq \{1, 2, \ldots, m\} = [m]$

costs $c_1, c_2, \ldots, c_n$

$I \subseteq \{1, 2, \ldots, n\}$ is a cover if $\bigcup_{i \in I} S_i = [m]$

Cost $c(I) = \sum_{i \in I} c_i$.

Simplified model of airline crew scheduling

$$x_j = \begin{cases} 0 : & j \notin I \\ 1 : & j \in I \end{cases}$$

$$\text{Total cost} = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$$

Subject to

$$i \in \bigcup_{j \in I} S_j , \forall i \quad : \quad \underbrace{a_{i1} x_1 + a_{i2} x_2 + \cdots + a_{in} x_n}_{\#\text{times } i \text{ appears in } S_j,\ j \in I} \geq 1$$

$$a_{ij} = \begin{cases} 0 : & i \notin S_j \\ 1 : & i \in S_j \end{cases}$$

DATA