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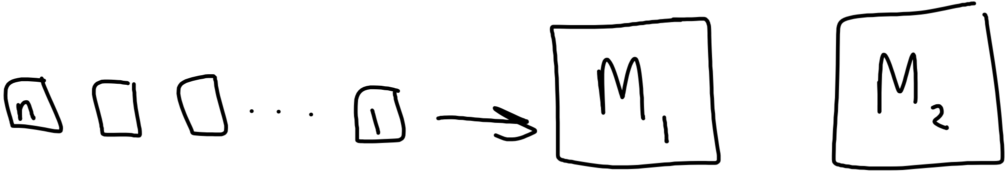


Job j: a_j on machine M_1

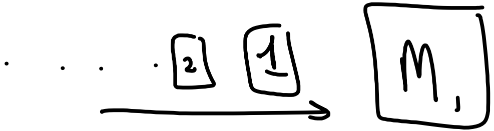
b_j on machine M_2

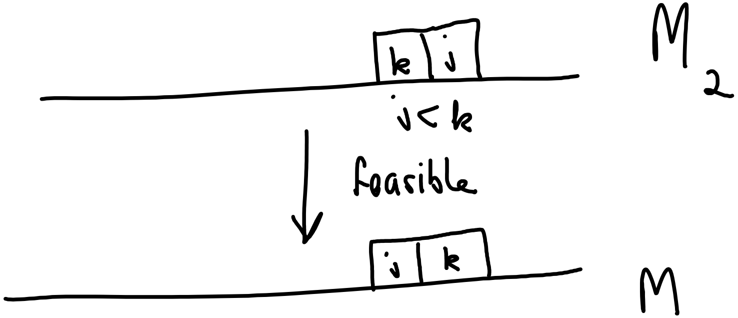
$A = \{j : a_j \leq b_j\}$

$B = \{j : a_j > b_j\}$

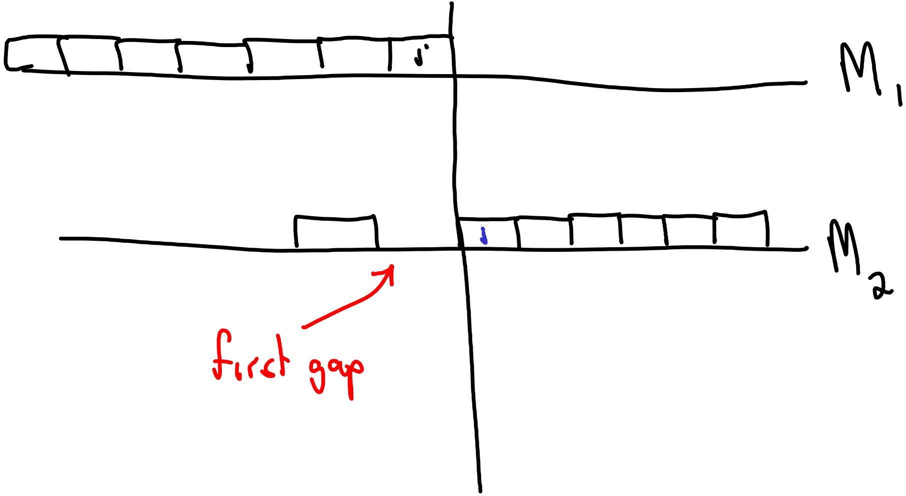
Johnson: A first in increasing a_j; B next in decreasing b_j

① We can assume a permutation schedule:





C_{max} does not increase



So if we reduce each a_j, b_j by δ: $a_j \leftarrow a_j - \delta \quad \forall j$

$b_j \leftarrow b_j - \delta \quad \forall j$

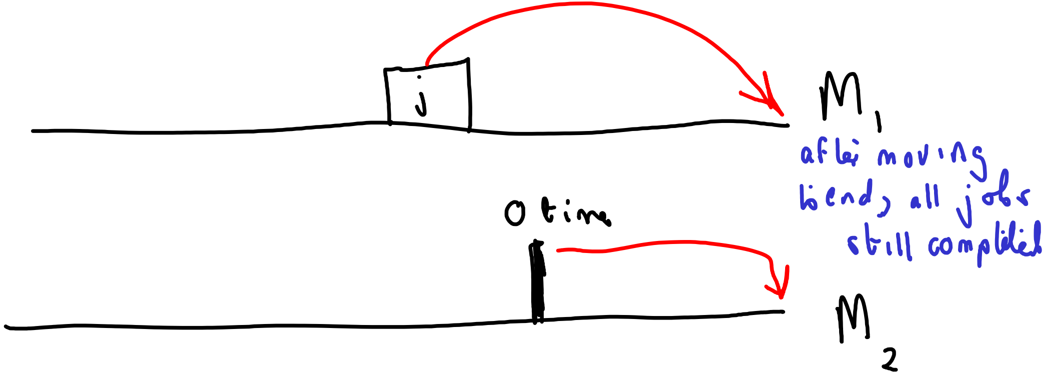
$$C_{max} \leftarrow C_{max} - (n+1)\delta$$

So optimum permutation is unchanged.

Choose $\delta = \min\{a_1, a_2, \dots, a_n, b_1, \dots, b_n\}$

Case 1: $\delta = a_j$

Case 2: $\delta = b_j$

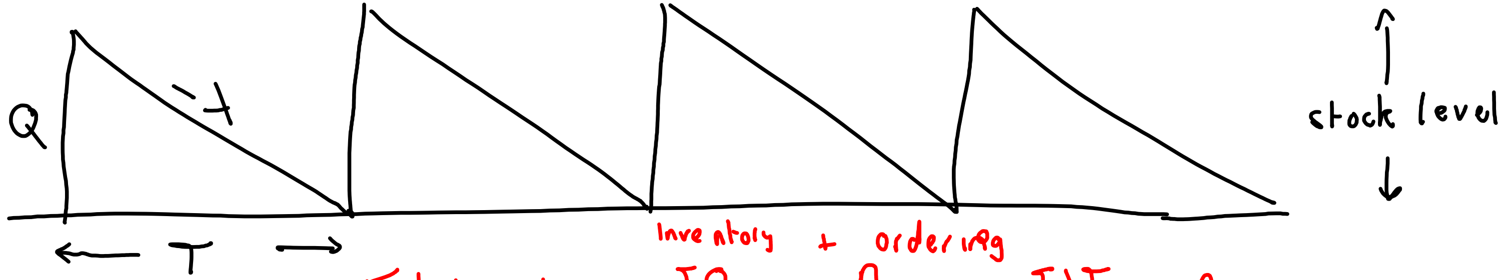


Inventory Control

Demand for your product is constant and λ per period.
At fixed intervals of time T we make an order of Q items. Problem is to minimise ordering + inventory cost.

A = fixed cost associated with an order.

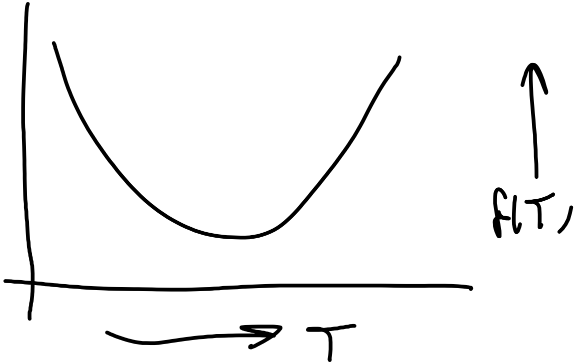
I = inventory cost per item per period.



inventory + ordering

$$\text{Total cost} = \frac{IQ}{2} + \frac{A}{T} = \frac{I\lambda T}{2} + \frac{A}{T}$$

Problem: minimise $\frac{I\lambda T}{2} + \frac{A}{T} = f(T)$



$$f'(T) = \frac{I\lambda}{2} - \frac{A}{T^2}$$

$$T = \sqrt{\frac{2A}{\lambda I}}$$

$$Q = \lambda T = \sqrt{\frac{2\lambda A}{I}}$$

Wilson Lot Size Formula.