Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 1: Due Monday September 10.

Q1 Solve the following knapsack problem:

$$\begin{array}{lrcrcrcl} \text{maximise} & 4x_1 & + & 8x_2 & + & 15x_3 & & \\ \text{subject to} & & & & & & & \\ & 3x_1 & + & 4x_2 & + & 5x_3 & \leq & 19 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

Q2 A county chairwoman of a certain political party is making plans for an upcoming presidential election. She has received the services of 10 volunteer workers for precinct work and wants to assign them to five precincts in such a way as to maximize their effectiveness. She feels that it would be inefficient to assign a worker to more than one precinct, but she is willing to assign no workers to any one of the precincts if they can accomplish more in other precincts.
The following table gives the estimated increase in the number of votes for the party's candidate in each precinct if it were allocated the various number of workers.

Number	Precinct				
of Workers	1	2	3	4	5
0	0	0	0	0	0
1	4	7	5	6	4
2	10	11	10	11	12
3	15	16	15	14	15
4	18	18	18	16	17
5	22	20	21	17	20
6	24	21	22	18	22
7	26	25	24	23	22
8	28	27	27	25	25
9	32	25	30	28	28
10	33	28	34	32	30

Use dynamic programming to find all solutions to the problem of maximising votes.

Q3 We are given $2n$ sets $D_1, D_2, \ldots D_n$ and $R_1, R_2, \ldots, R_n$ where n is even. Also, $|D_i| + |R_i| = m$ for $i = 1, 2, \ldots, n$. Find an algorithm that will check to see if the following is possible: Find a set $I \subseteq [n]$, $|I| = n/2$ such that

$$\sum_{i \in I} |D_i| \geq \sum_{i \in I} |R_i| \text{ and } \sum_{i \notin I} |D_i| \geq \sum_{i \notin I} |R_i|.$$

Your algorithm should run in time polynomial in m, n.