9\7\12

Knapsack problem:

$$f_r(w) = \max_{x_r} \left[c_r x_r + f_{r-1}(w - w_r x_r) \right]$$

Time to compute $f_n(W) = O(nW^2)$

Reduce to $O(nW)$

choices for r

choices for w × time to compute

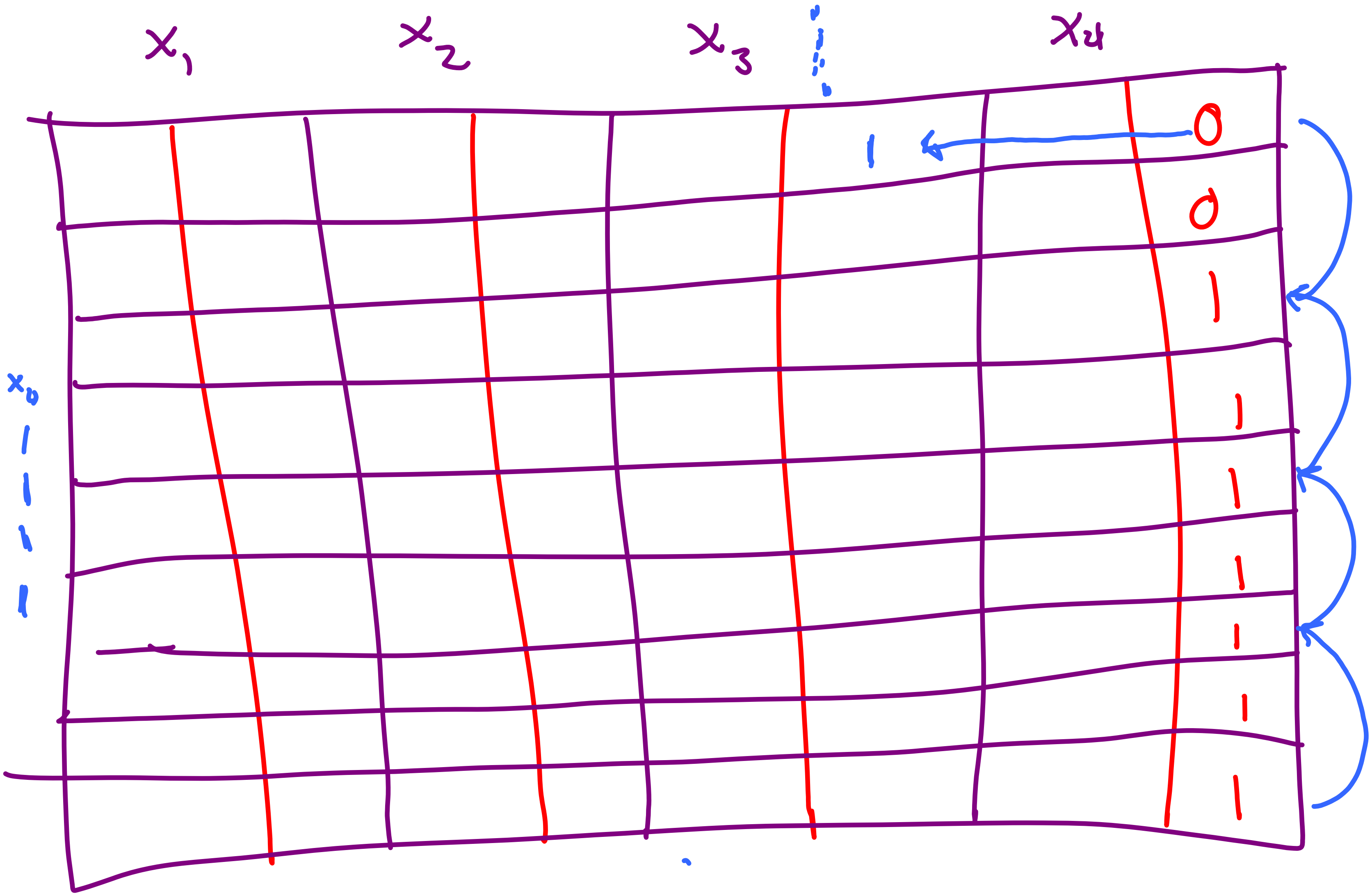
$$f_r(w) = \max \begin{cases} f_{r-1}(w) & x_r = 0 \\ c_r + f_r(w - w_r) & x_r \geq 1 \end{cases}$$

Maximise $x_1 + 4x_2 + 6x_3 + 8x_4$

x_1	x_2	x_3	x_4
0	0	1	1

$$x_1 + 3x_2 + 4x_3 + 5x_4 \leq 9$$

w	f_1	b_1	f_2	b_2	f_3	b_3	f_4	b_4
0	0	0	0	0	0	0	0	0
1	1	1	1	0	1	0	1	0
2	2	1	2	0	2	0	2	0
3	3	1	4	1	4	0	4	0
4	4	1	5	1	6	1	6	0
5	5	1	6	1	7	1	8	1
6	6	1	8	1	8	0/1	9	1
7	7	1	9	1	10	1	10	1
8	8	1	10	1	12	1	12	1
9	9	1	12	1	13	1	14	1



x_1 x_2 x_3 x_4

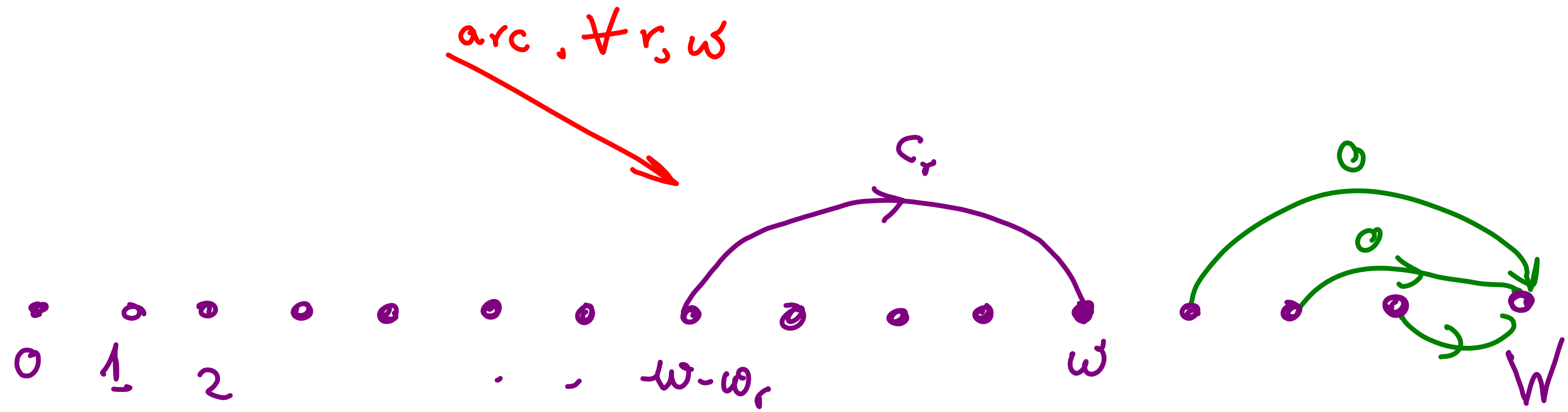
0 1

1

1

1

$\omega_4 = 2$



Problem: find longest path in digraph from 0 to W.

$$\ell(w) = \max \text{ length path to } w$$

$$= \max_r \; \ell(w - w_r) + c_r$$