9/19/12

Markov decision processes

D_i = {possible "decisions"}

If you choose $d \in D_i$ then

i

$P_d(i, j)$

$= P$[going to j next]

j

α

Combinatorial Optimisation

1. Shortest paths
2. Assignment problem
3. Spanning Tree

Shortest Paths

Digraph $D = (V, A)$

Length $l : A \longrightarrow \mathbb{R}$ [edge a has length $l(a)$]

If P is a path then

$$l(P) = \sum_{a \in P} l(a)$$

Single Source Problem

Given $s \in V$ find a shortest path from s to all $v \in V$.

Case 1

Non-negative arc lengths

$$\ell(a) \geq 0 , \quad \forall a \in A$$

Dijkstra's Algorithm

Initially we have $d(s) = 0$ & $d(v) = \infty$ for all $v \neq s$.

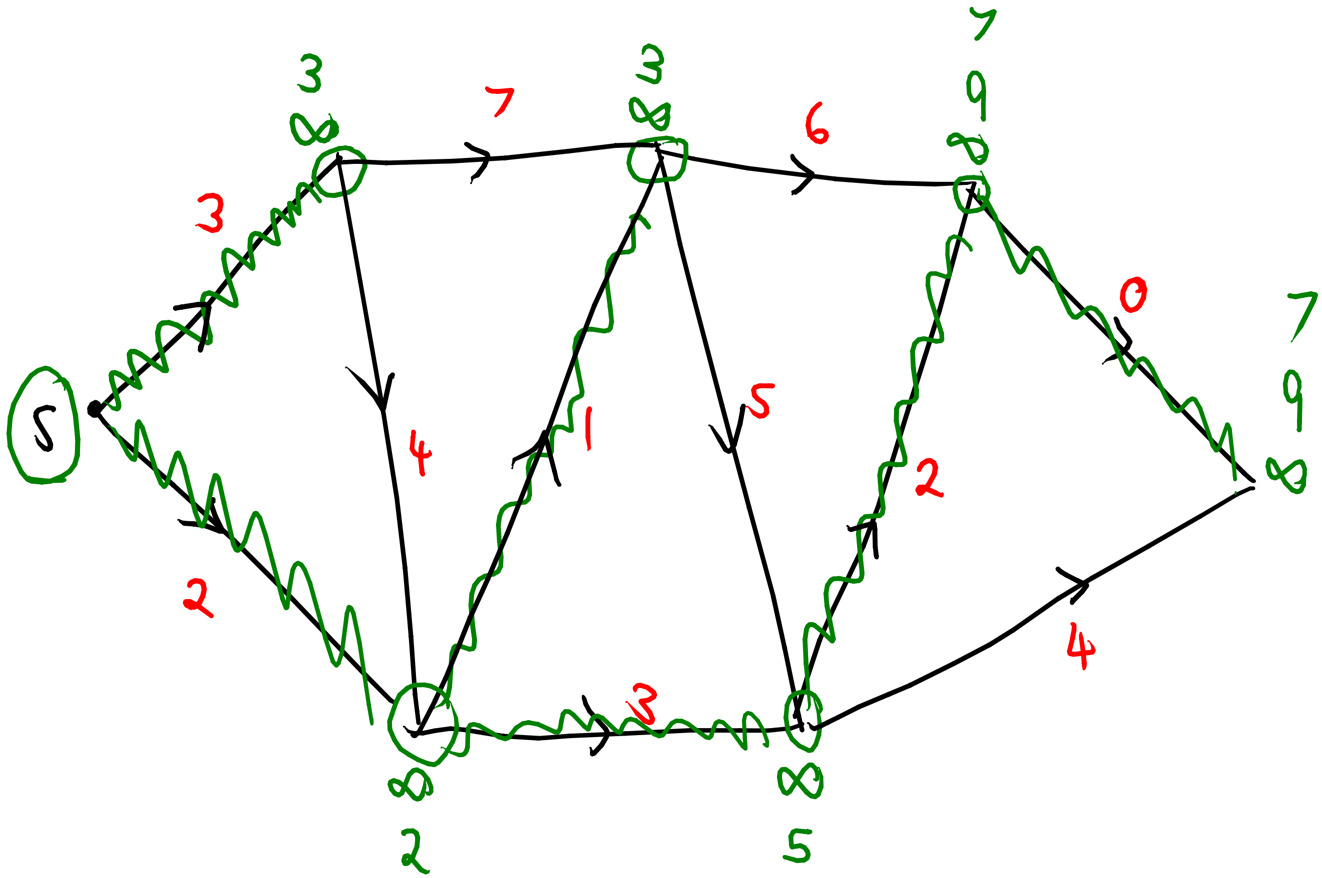
[$d(v)$ is our current estimate of the shortest distance from s to v]

$S = \emptyset$ We know shortest path from s to $v \in S$

$x \leftarrow s$

repeat

$$\left\{ \begin{array}{l} \text{process } x : \forall v \notin S ;\ d(v) \leftarrow \min\{d(v), d(x) + \ell(x,v)\} \\ S = S \cup \{x\} \\ d(y) = \min\{v \notin S : d(v)\} \\ x \leftarrow y \end{array} \right.$$


S
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5

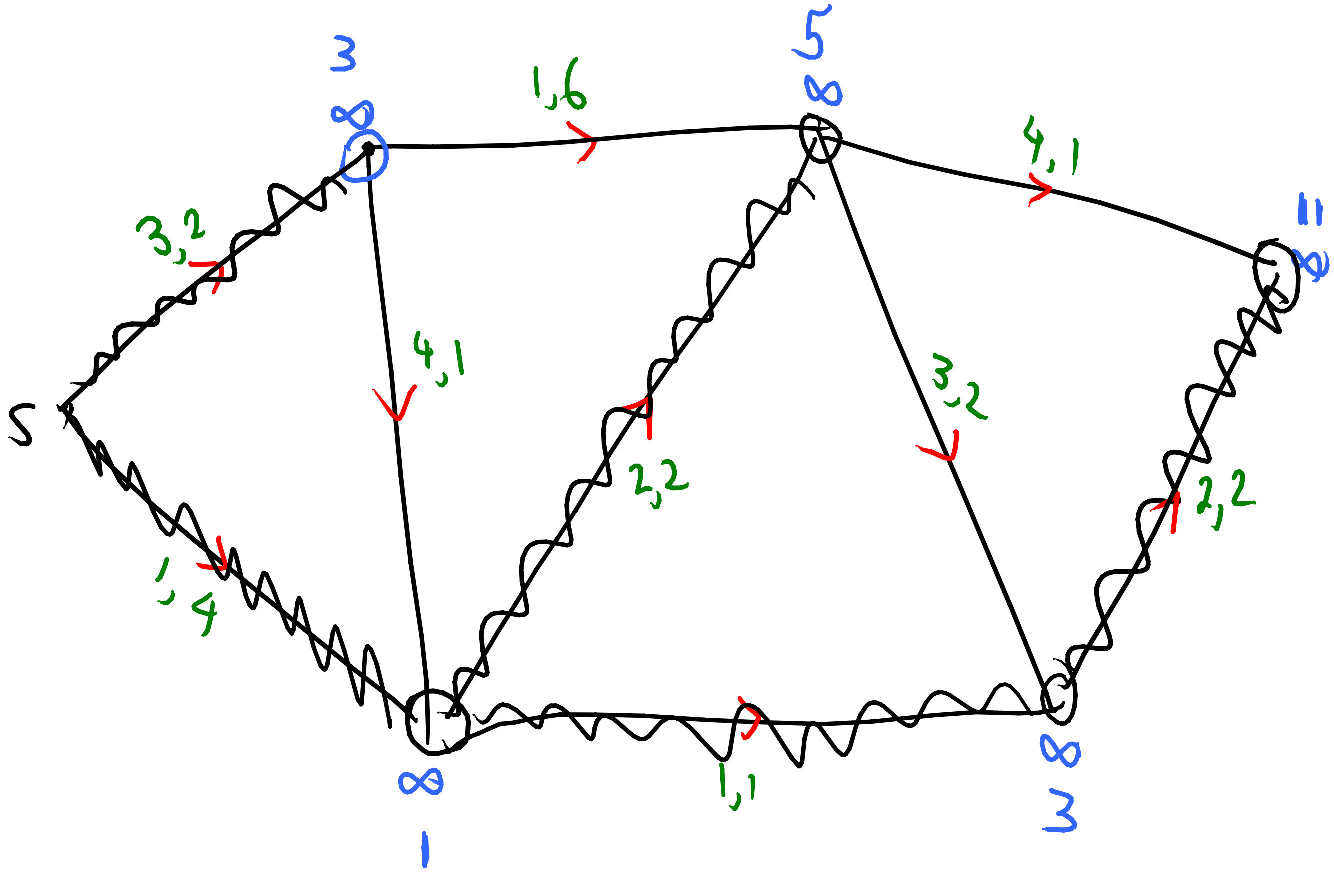
Time dependent arc lengths

$$l : A \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+$$

$l(a,t)$ = length of arc $a = (x,y)$ if we arrive at x at time t.

$P = (u_0, u_1, u_2, \ldots)$: start at $t = 0$

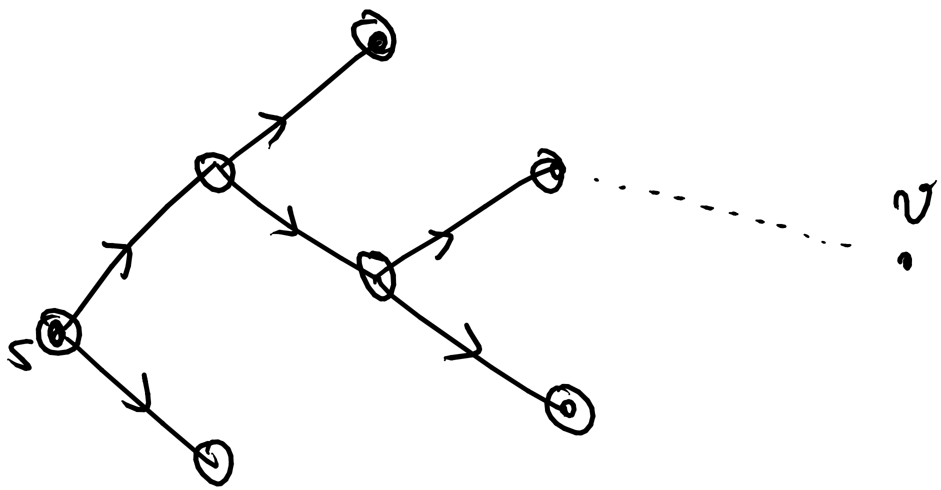
$$l(P) = \underbrace{l((u_1,u_2),0)}_{\xi_1} + \underbrace{l((u_2,u_3),\xi_1)}_{\xi_2} + l((u_3,u_4),\xi_1+\xi_2) + \ldots$$



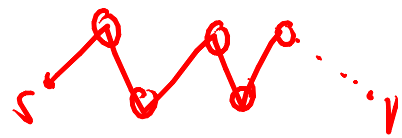
$$\ell((x,y),t) = \alpha + \beta t \quad \text{for some } \alpha, \beta$$

Claim: $d(v)$ is correct for all $v \in S$

Proof: by induction on $|S|$.



$d(v)$ = minimum length of a path from s to v of the form



Any other path s to v

not nec. tree path $\geq d(w) \geq d(v)$. $\square$