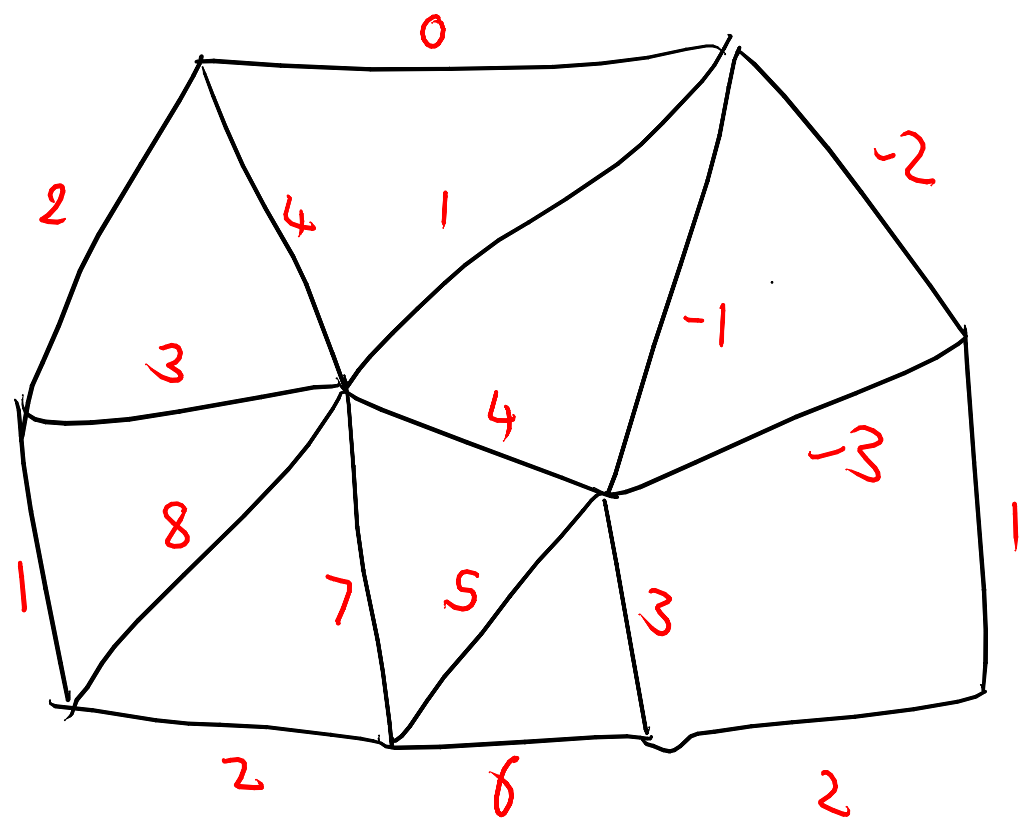
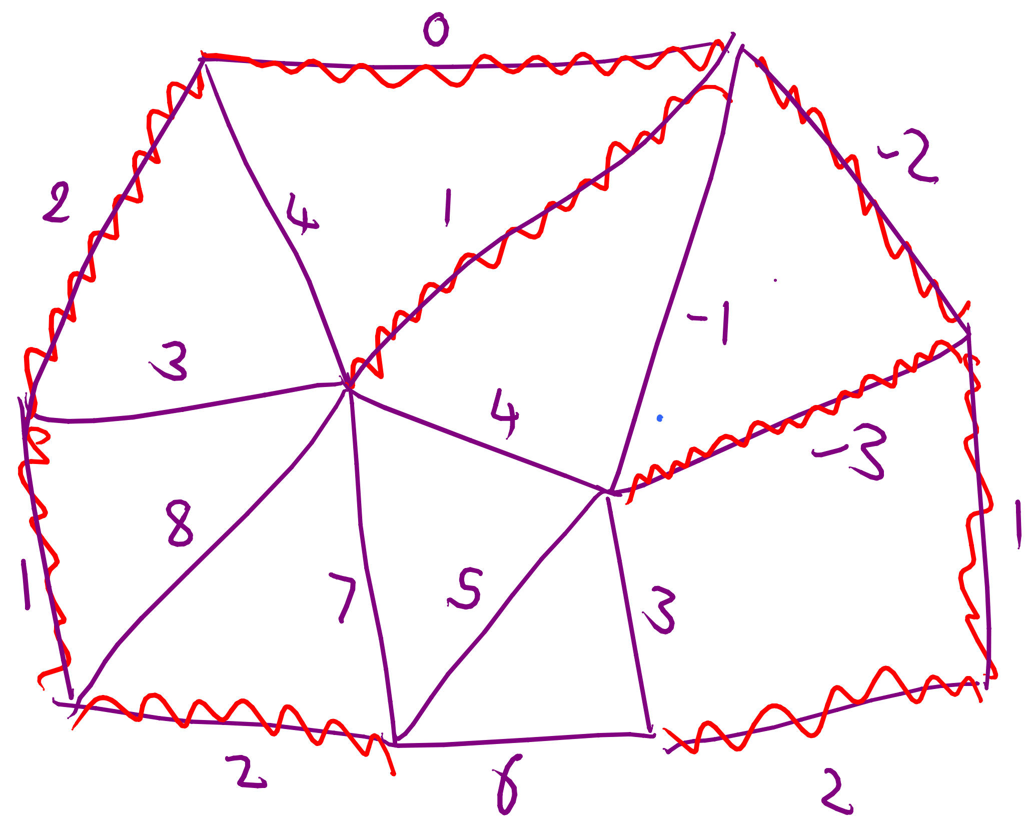
5/10/12

Minimum Spanning Tree

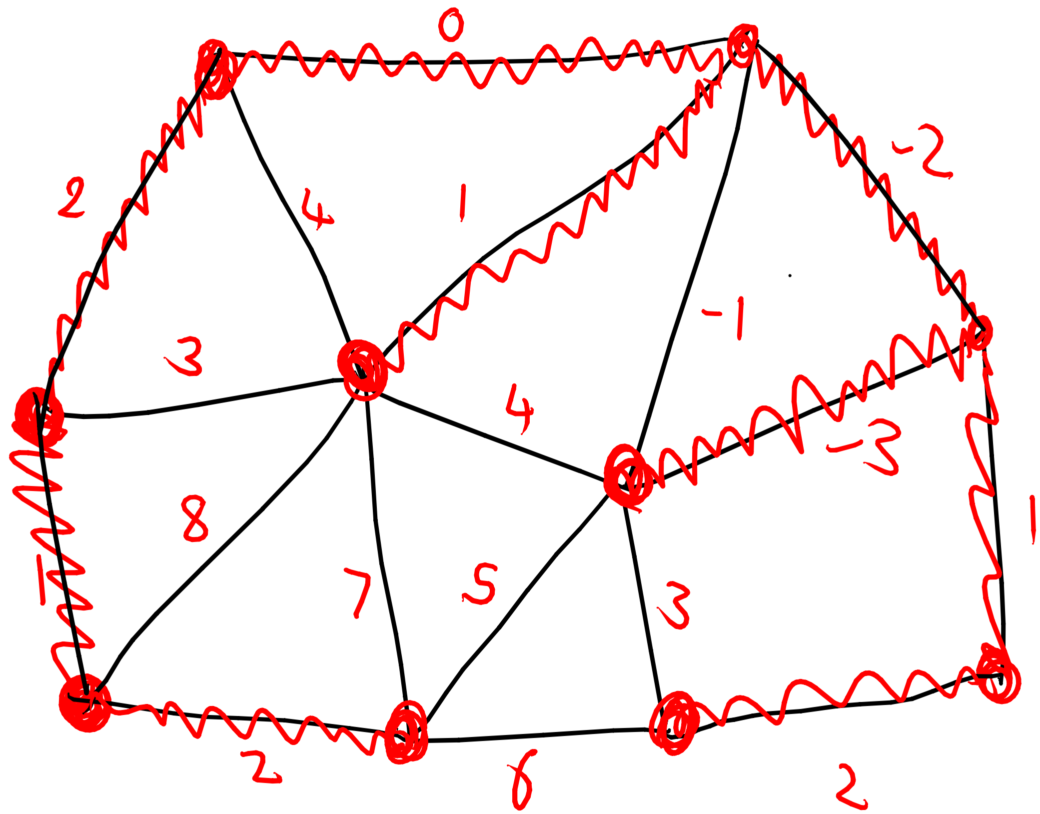
* Connected subgraph without cycles,



Problem: find minimum length spanning tree*



Greedy Algorithm — Kruskal



Dijkstra's algorithm

More general algorithm

Suppose at some time we have selected forest

(i) Choose a component C

(ii) Add shortest edge with <u>one</u> vertex in C

Claim: at all times $\exists$ a minimum length tree that contains the edges chosen so far

Proof

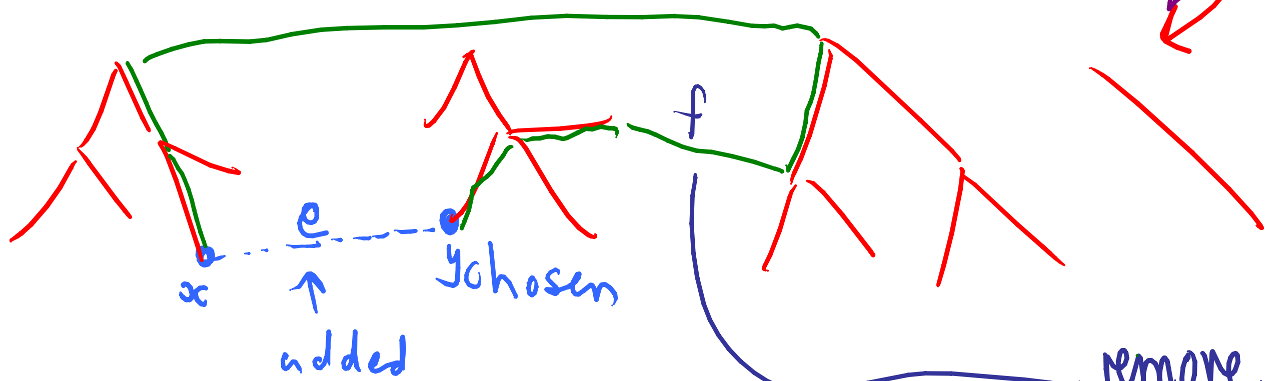
By induction.

True when we have chosen 0 edges.

Suppose we have chosen k edges and $\exists$ a shortest tree containing selected edges

Selected edges $\subseteq T$ = some shortest tree $\supseteq$ wg

Replace T by T+e−f



remove. It is no shorter than e

If $e \in T$ then T will suffice. Suppose $e \notin T$ ∴ T contains a path from x to y.

Integer Programming

A is $m \times n$

Minimise $\underline{c}^T \underline{x}$

s.t. $A\underline{x} = \underline{b}$

$\underline{x} \geq 0$

x_j is integer for $j \in J \subseteq \{1, 2, \ldots, n\}$

Example

(1) Knapsack problem

Maximize $p_1 x_1 + \cdots + p_n x_n$

s.t. $w_1 x_1 + \cdots + w_n x_n \le W$

$x_j \ge 0$

x_j integer for $j = 1, 2, \ldots, n$

2/ n projects could undertaken. [n investments]

If activated, project j will yield p_j in profit.

There are m resources.

Project j requires a_{ij} units of resource i

r_i units of resource i are available

Problem: select k projects to maximise profit

$$x_j = \begin{cases} 0 & \text{do not carry out project } j \\ 1 & \text{" \quad " \quad " \quad "} \end{cases}$$

Maximise

$$\text{Profit} = p_1 x_1 + p_2 x_2 + \cdots + p_n x_n$$

$$x_1 + x_2 + \cdots + x_n = k$$

$$a_{i1} x_1 + a_{i2} x_2 + \cdots + a_{in} x_n \leq r_i \quad \forall i$$

$0 \leq x_j \leq 1$ & $\underline{x_j \text{ integer}}$