10\3\12

Suppose we have min. cost assignment

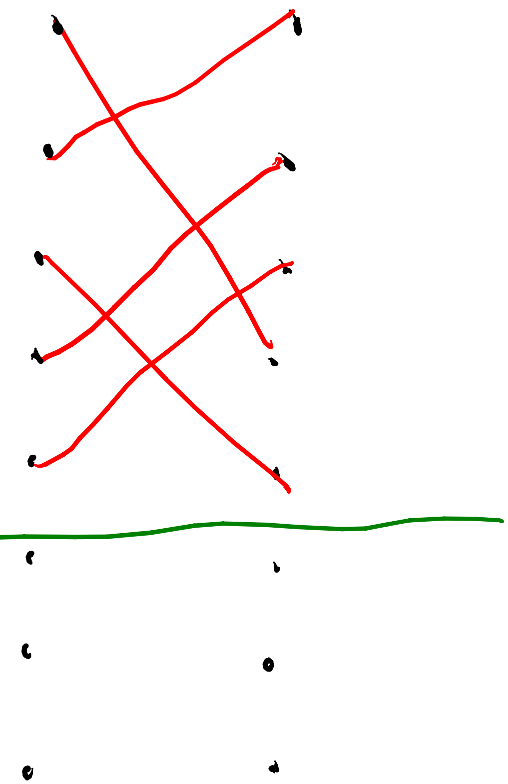
$g \; [k] \longrightarrow [k]$.

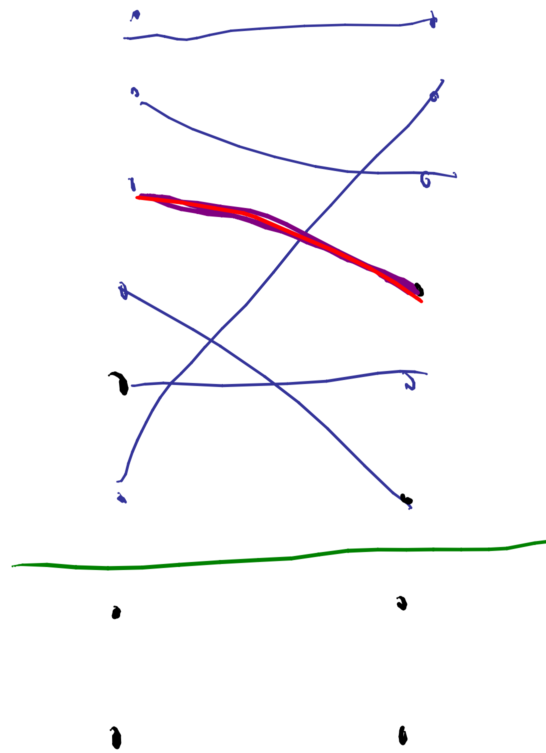
How do we get min. cost assignment

of $[k+1] \longrightarrow [k+1]$?

min. $[n] \to [k]$

M_k

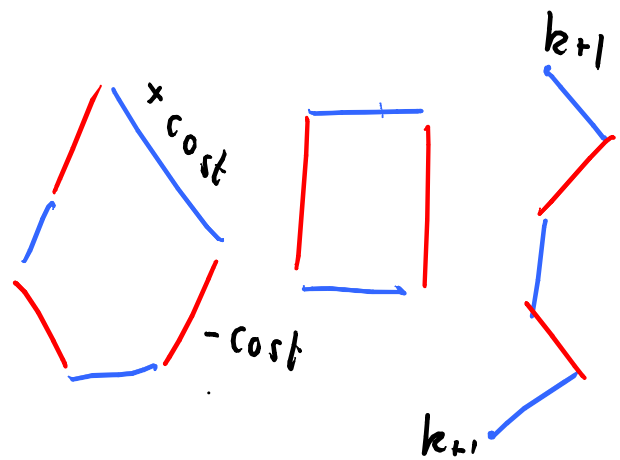


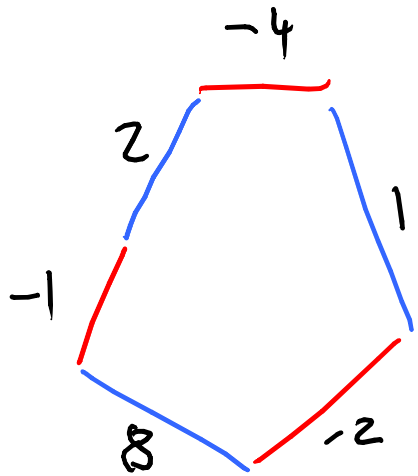


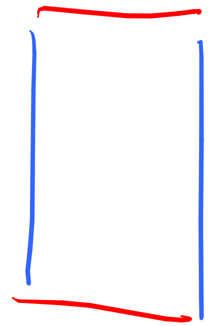
M

$M \oplus M_k \longrightarrow$

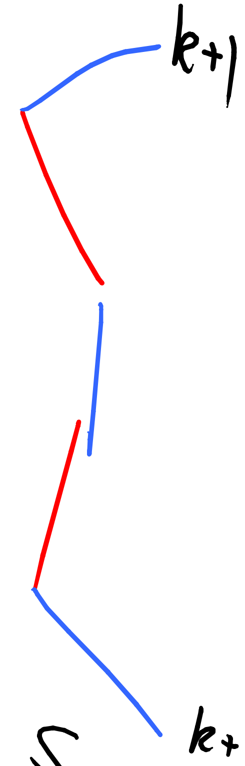
$(M \setminus M_k) \cup (M_k \setminus M)$







Same here



Same here

$8+1+2-(2+4+1)$

= change in cost

if I replace

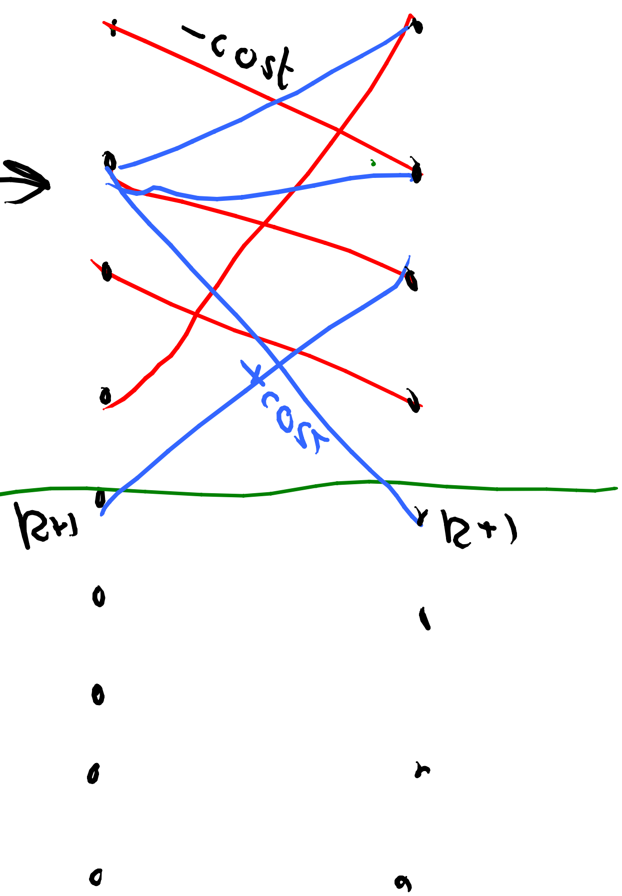
by

For a cycle sum of red costs $\geq$
sum of blue costs
else M_k is not minimal

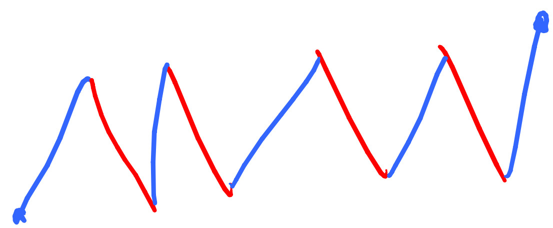
Therefore choose M with no cycles & shortest length

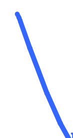
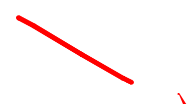
Algorithm

only some of edges, to avoid mess

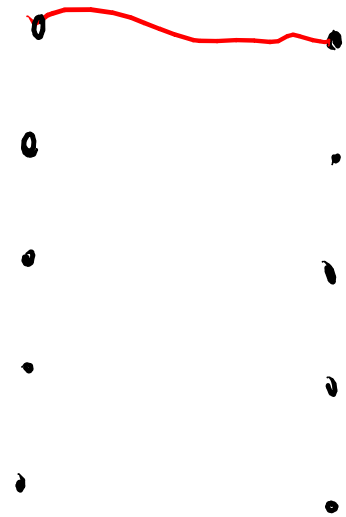


Find shortest path from (k+1) to (k+1)



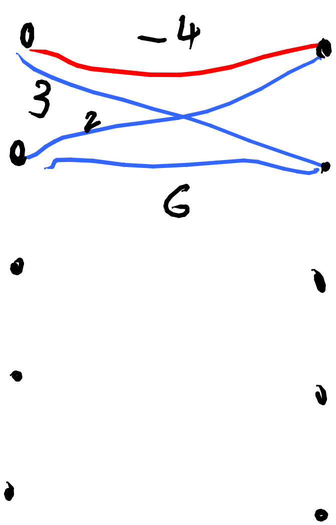
Add  and delete 

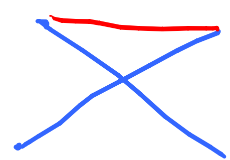
$$\begin{bmatrix} 4 & 3 & 1 & 4 & 5 \\ 2 & 6 & 4 & 3 & 1 \\ 3 & 2 & 1 & 5 & 6 \\ 1 & 4 & 3 & 2 & 5 \\ 2 & 3 & 1 & 4 & 2 \end{bmatrix}$$



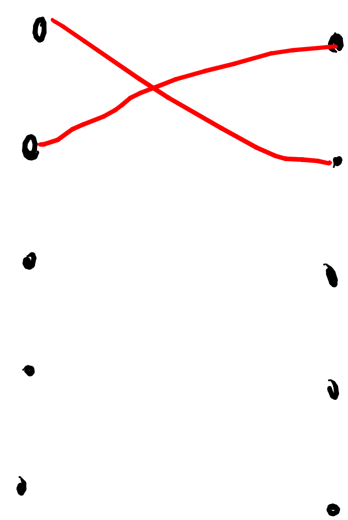
$k=1$

$k=2$



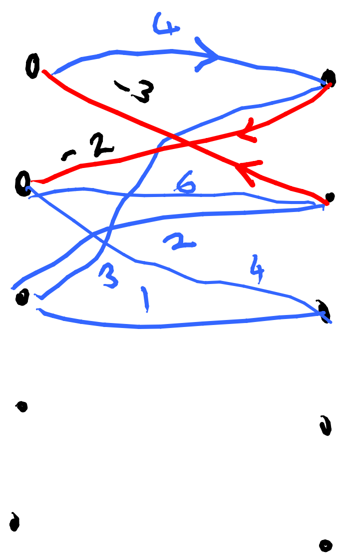


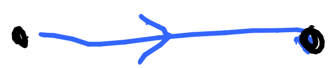
Shortest



$$\begin{bmatrix} 4 & 3 & 1 & 4 & 5 \\ 2 & 6 & 4 & 3 & 1 \\ 3 & 2 & 1 & 5 & 6 \\ 1 & 4 & 3 & 2 & 5 \\ 2 & 3 & 1 & 4 & 2 \end{bmatrix}$$

$k=2$





Shortest

Total running time

$= O(n \times n^3) = O(n^4)$

Linear Programming

Minimise $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij}$

s.t.

$\sum_{i=1}^{n} x_{ij} = 1 \quad \forall j$

$\sum_{j=1}^{n} x_{ij} = 1 \quad \forall i$

Every basic feasible solution to LP with $0 \leq x_{ij} \leq 1$ satisfies

replace by

$\boxed{x_{ij} = 0 \text{ or } 1}$

$x_{ij} = 1$