10/17/12

$$\begin{bmatrix} \\ a_{ij} \\ \\ \end{bmatrix} \Longrightarrow \begin{array}{cc} & v \\ u & \begin{bmatrix} \\ \text{PAY}(u,v) \\ := \text{average payoff} \\ \\ \end{bmatrix} \end{array}$$

A strategy for A is a sequence

$i_1, i_2, i_3, \ldots$

pure if $i_1 = i_2 = \ldots$

and B, a sequence

$j_1, j_2, j_3, \ldots$

can be random

$$P_A = A\text{'s gurantee} = \max_{u} \text{ROWMIN}(u)$$

$$P_B = B\text{'s gurantee} = \min_{v} \text{COLMAX}(v)$$

$$P_A \leq P_B$$

Stable Solution:

$$\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

(u_0, v_0)

$$\text{PAY}(u, v_0) \leq \text{PAY}(u_0, v_0) \leq \text{PAY}(u_0, v)$$

NASH EQUILIBRIUM

$P_A = -1$

$P_B = 1$

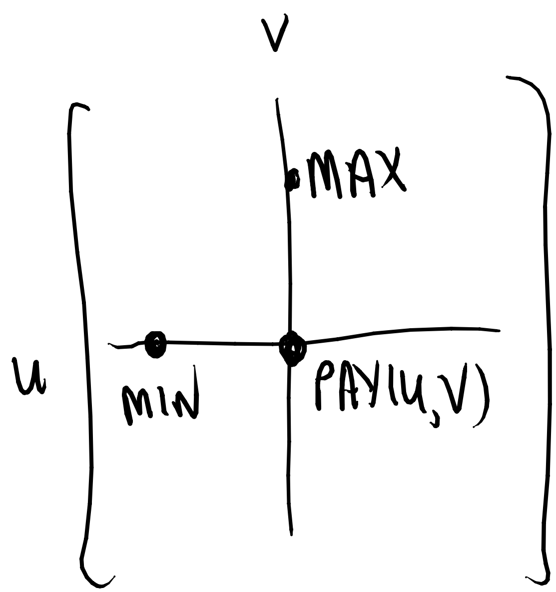
Thm

(i) $P_A \le P_B$

(ii) $P_A = P_B$ if $\exists$ stable solution

Proof

(i)



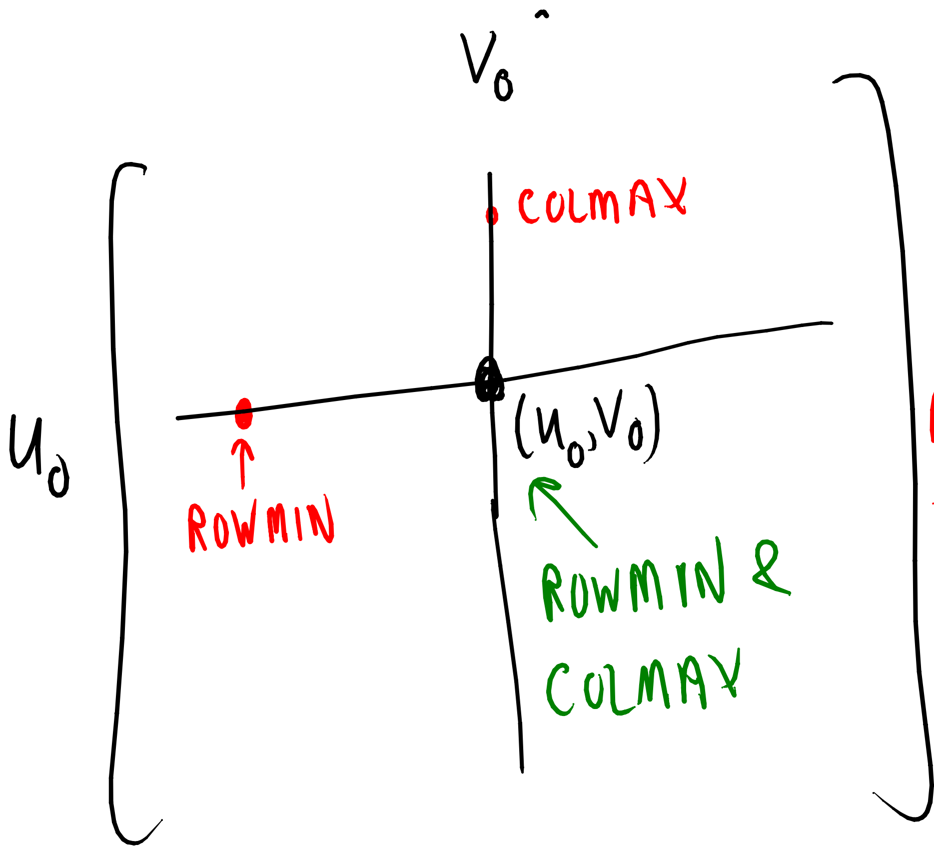
$$\mathrm{ROWMIN}(u) \le \mathrm{PAY}(u,v) \le \mathrm{COLMAX}(v)$$

largest ROMIN $\le$ smallest COLMAX

$$\Downarrow$$

$$P_A \le P_B$$

(ii) Suppose that $\underset{\uparrow\ \mathrm{ROWMIN}(u_0)}{P_A} = \underset{\uparrow\ \mathrm{COLMAX}(v_0)}{P_B}$



$$\begin{aligned} PAY(u_0, V_0) &\leq COLMAX \\ &= P_B \\ &= P_A \\ &= ROWMIN(u_0) \end{aligned}$$

$$\underline{PAY(u_0, V_0) = ROWMIN(u_0)}$$

$$\begin{aligned} PAY(u_0, V_0) &\geq ROWMIN \\ &= P_A \\ &= P_B \\ &= COLMAX(v_0) \end{aligned}$$

$$PAY(u_0, V_0) = COLMAX(v_0)$$

Suppose (u_0, v_0) is Stable $\Rightarrow$ $PAY(u_0, v_0) = ROWMIN(u_0) = COLMAX(v_0)$

$$P_A \geq ROWMIN(u_0)$$
$$= COLMAX(v_0)$$
$$\geq P_B$$

Problem: Enlarge set of stratgies so that $P_A = P_B$

"Mixed Strategy"

$[m \times n]$ A choose $\underline{p}$ $p_1, p_2, \ldots, p_m \geq 0$

$p_1 + \cdots + p_m = 1$

B chooses $\underline{q}$

$q_1, q_2, \ldots, q_n \geq 0$

$q_1 + \cdots + q_n = 1$

A plays i with probability p_i

B plays j with probability q_j

$$\text{PAY}(\underline{p}, \underline{q}) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} p_i q_j$$

Expected winnings for A

<u>Theorem</u>

$P_A = P_B$ with this set of strategies