Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 1: Due Wednesday September 14.

Q1 Solve the following knapsack problem:

$$\begin{array}{lrcrcrcl} \text{maximise} & 5x_1 & + & 8x_2 & + & 15x_3 & & \\ \text{subject to} & & & & & & & \\ & 3x_1 & + & 4x_2 & + & 5x_3 & \leq & 19 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

Q2 Consider a 2-D map with a horizontal river passing through its center. There are n cities on the southern bank with x-coordinates $a(1)...a(n)$ and n cities on the northern bank with x-coordinates $b(1)...b(n)$. You want to connect as many north-south pairs of cities as possible with bridges such that no two bridges cross. When connecting cities, you can only connect city i on the northern bank to city i on the southern bank. Construct a Dynamic Programming solution to this problem. (You can assume that $a(1) < a(2) < \cdots < a(n)$, but you **cannot** assume that $b(1) < b(2) < \cdots < b(n)$. If both sequences are increasing, then the problem is trivial).

Q3 We are given $2n$ sets $D_1, D_2, \ldots D_n$ and $R_1, R_2, \ldots, R_n$ where n is even. Also, $|D_i| + |R_i| = m$ for $i = 1, 2, \ldots, n$. Find an algorithm that will check to see if the following is possible: Find a set $I \subseteq [n]$, $|I| = n/2$ such that

$$\sum_{i \in I} |D_i| \geq \sum_{i \in I} |R_i| \text{ and } \sum_{i \notin I} |D_i| \geq \sum_{i \notin I} |R_i|.$$

Your algorithm should run in time polynomial in m, n.