9\26\11

All pairs shortest path problem.

Given matrix $D = \|d_{ij}\|$ where

d_{ij} = length of the edge (i,j).

Find shortest path between every pair of vertices.

Assume there are no negative cycles.

Floyd's Algorithm.

For $k = 1$ to n do

For $i = 1$ to n do

For $j = 1$ to n do

$D_{i,j} = \min\{D_{i,j}, D_{i,k} + D_{k,j}\}$

od

od

od

Claim: After first l steps $k = 1, 2, \ldots, l$

$D_{i,j}$ = min. length of a path $i \to j$ with internal vertices in $\{1, 2, \ldots, l\}$.

D_{ij} = min. length of such a path. (walk)

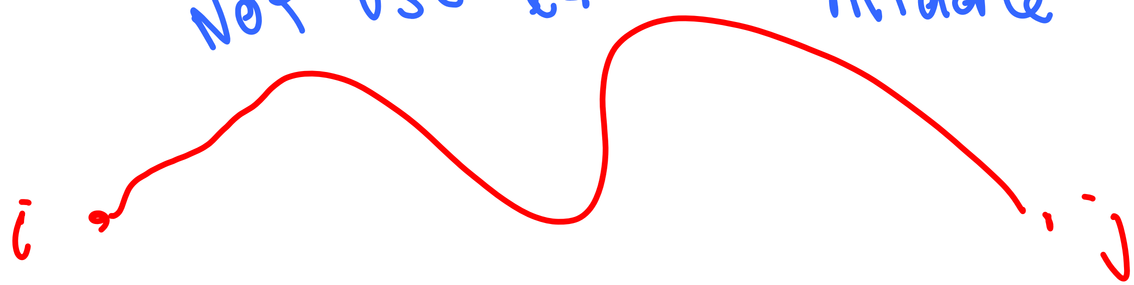
Proof: By induction on ℓ.

Base Case: $\ell = 0$. A path from $i \to j$ using interior vertices from $\underbrace{1, 2, \ldots, 0}_{\emptyset}$ must be the arc (i, j)

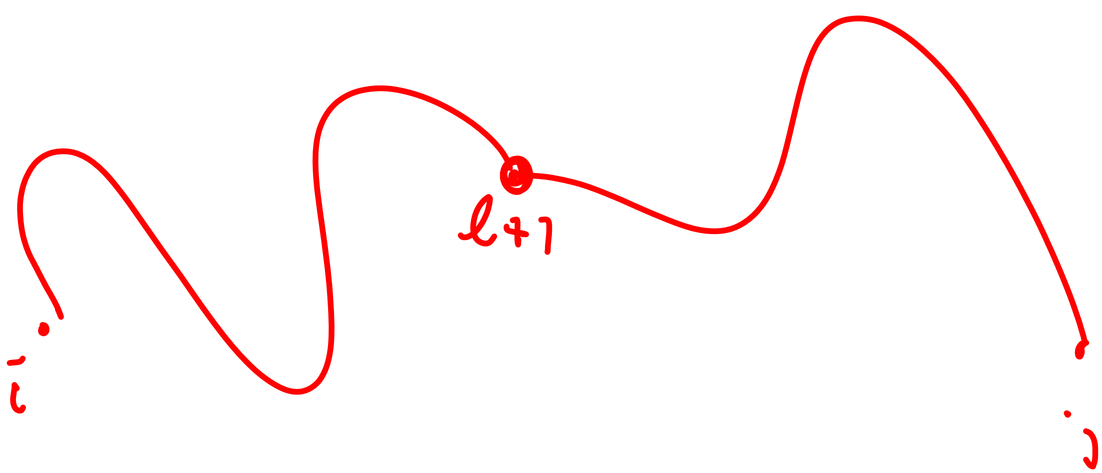
Inductive Step: Assume true for some $\ell \geq 0$

Paths from $i \longrightarrow j$ using $1, 2, \ldots, \ell, \ell+1$.

Not use $\ell+1$ in middle



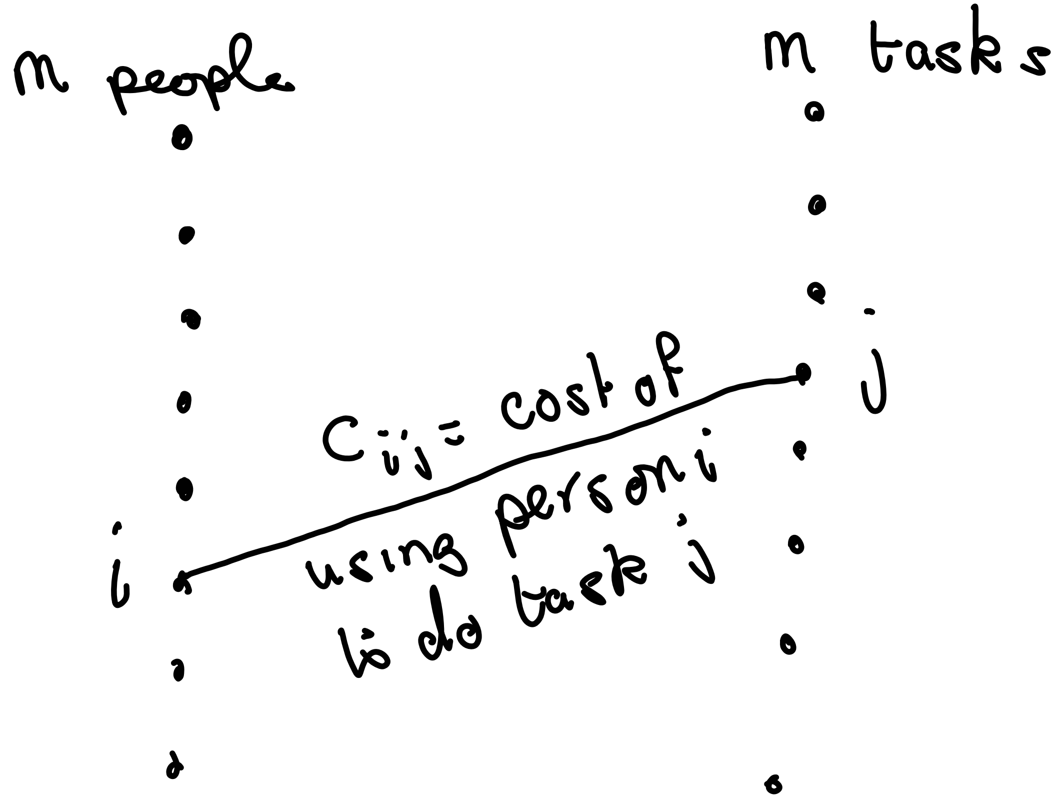
Min. length of these is D_{ij} [induction]



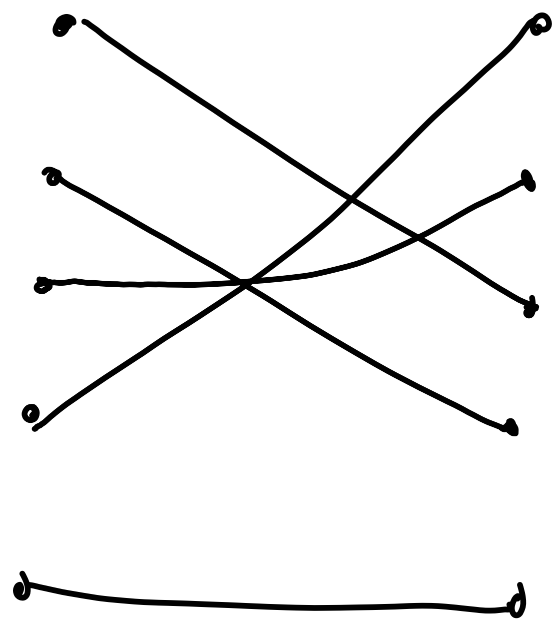
Min. length of these is $D_{i,\ell+1} + D_{\ell+1,j}$

Floyd take min. of these.

Assignment Problem



Problem: Assign one task to each person to minimise total cost.



Perfect matching of minimum total cost

choices for M : n!

Idea

For $k = 1$ to n do

given min. cost matching from

$[k] \longrightarrow [k]$

set up problem of extending matching

to $[k+1] \longrightarrow [k+1]$ as a shortest

path problem.

$$\left([k] \rightarrow [k]\right) \xrightarrow[\text{s.p. prob.}]{} \left([k+1] \rightarrow [k+1]\right)$$

k=5

Red is h→k minimal