9\19\11

## Optimality Condition

$\Pi$ is optimal iff

$$y_i = \min_j \left[ c(i,j) + \alpha y_j \right] \quad (*)$$

**Proof**

Assume first that $(*)$ is true.

Choose any other policy $\hat{\pi}$ and $\hat{y}$ denotes the values.

To show: $y_i \leq \hat{y}_i$, $\forall i \in V$.

$$\hat{y}_i = c_{i,\hat{\pi}(i)} + \alpha \hat{y}_{\hat{\pi}(i)}, \quad \forall i \in V$$

$$y_i \leq c_{i,\hat{\pi}(i)} + \alpha y_{\hat{\pi}(i)}$$

$$\underbrace{\hat{y}_i - y_i}_{\xi_i} \geq \underbrace{\alpha\left(\hat{y}_{\hat{\pi}(i)} - y_{\hat{\pi}(i)}\right)}_{\xi_{\hat{\pi}(i)}}$$

$$\xi_i \geq \alpha\, \xi_{\hat{\pi}(i)} \qquad \forall i$$

$$\geq \alpha^2\, \xi_{\hat{\pi}^2(i)}$$

$$\vdots$$

$$\geq \alpha^k\, \xi_{\hat{\pi}^k(i)} \qquad \forall k$$

But $\alpha^k \to 0$ and so $\xi_i \geq 0$

Suppose $(*)$ is not true:

Define $\tilde{\pi}$ by

$$c_{i,\tilde{\pi}(i)} + \alpha y_{\tilde{\pi}(i)} = \min_j \left[ c_{ij} + \alpha y_j \right]$$

$$\tilde{y}_i = c_{i,\tilde{\pi}(i)} + \alpha \tilde{y}_{\tilde{\pi}(i)}$$

$$\boxed{\xi_i = \tilde{y}_i - y_i}$$

$$y_i \geq c_{i,\tilde{\pi}(i)} + \alpha y_{\tilde{\pi}(i)}$$

$>$ for $i \in I \neq \emptyset$

$$\xi_i \leq \alpha \, \xi_{\pi(i)}$$

$< \quad i \in I$

$$\Rightarrow \quad \xi_i \leq 0 \quad \& \quad \xi_i < 0 \text{ for } i \in I.$$

# Linear Programming

$$y_i = \min_j \left[ c(i,j) + \alpha y_j \right] \quad (*)$$

Define an LP in the $y_i$'s.

Constraints:

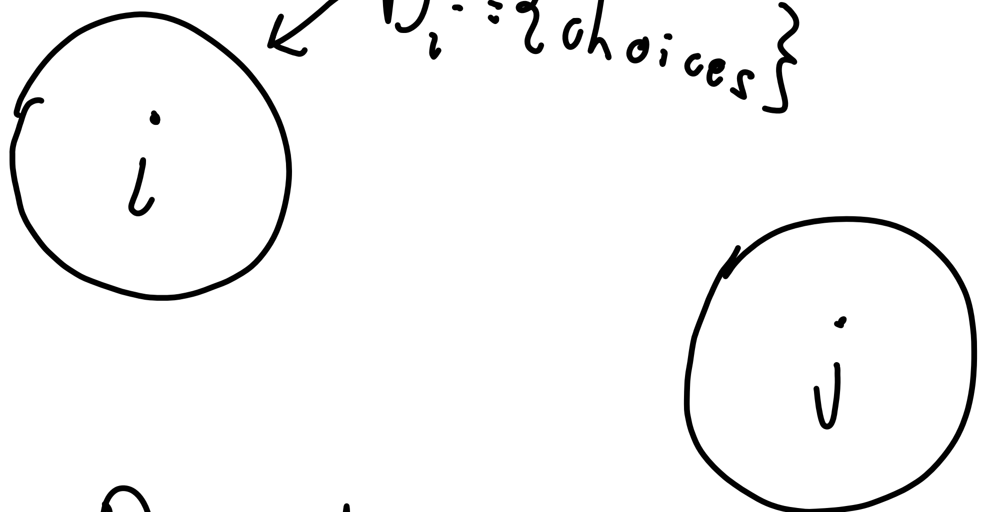
$$y_i - \alpha y_j \leq c_{i,j} \quad \forall i,j$$

Maximise $y_1 + y_2 + \cdots + y_n$ s.t

$(*)$ holds for optimum solution to

## Add probability

$D_i := \{\text{choices}\}$




for $x \in D_i$: we have

(i) $P(x, i, j)$ = Prob. of reaching $j$

(ii) $C(x, i)$ = expected cost of first move.

Once again we must choose a fix $x = \pi(i)$ for each $i \in V$.