9\14\11

Planning over an infinite horizon.

Consider production where $n = \infty$

Demand = 3 per period. H = max inv. = 5

Feasible production schedule:

3	3	3	3	3	3	3
4	2	4	2	4	2	4
5	1	5	1	5	1	5

$c(3) = 7$

$c(4) = 10$, $c(2) = 3$

$c(5) = 11$, $c(1) = 2$

Net present value:

assume a discount factor $\alpha < 1$.

Suppose we have costs

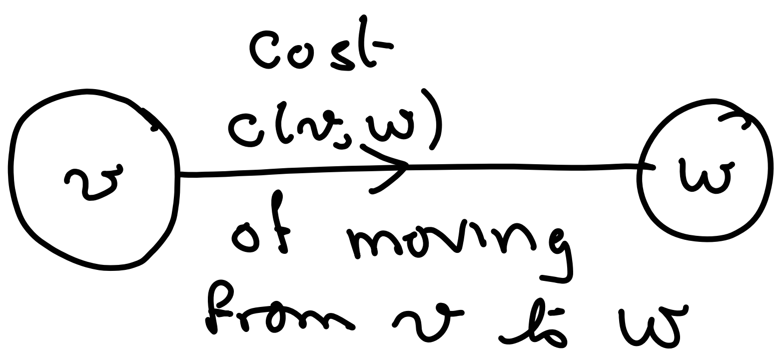
$c_0, c_1, c_2, c_3, \ldots$

$$N.P.V. = c_0 + \alpha c_1 + \alpha^2 c_2 + \ldots$$

Now we a consider a situation where we minimise N.P.V.

System: $V = \{\text{states}\}$

Production problem: $V = \{0, 1, 2, \dots, H\}$



start in some state i and take an infinite sequence of state changes

$V \times V$ matrix $\quad C = \| c(v,w) \|$

$H = 3$

$c(x) = x^2$

$d = 3$

$$\begin{array}{c|c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 9 & 16 & 25 & 36 \\ 4 & 9 & 16 & 25 \\ 1 & 4 & 9 & 16 \\ 0 & 1 & 4 & 9 \end{bmatrix} \end{array}$$

Goal: find sequence that minimises N.P.V.

Problem: start in v_0, find sequence

$$v_0, v_1, v_2, v_3, \ldots$$

that minimises

$$c(v_0, v_1) + \alpha\, c(v_1, v_2) + \alpha^2 c(v_2, v_3) + \cdots$$

∞ set of possible sequences.

How do we reduce to a finite # of choices.

Possible sequence

$\sigma_1 = 3, 8, 2, 5, 6, 3, 7, 4, 5, 7, 2$

What is wrong with this.

Consider

$\sigma_2 = 3, 7, 4, 5, 7, 2, \ldots\ldots$

If $NPV(\sigma_2) > NPV(\sigma_1)$ then

$\sigma_3 = 3, 8, 2, 5, 6, \sigma_1$ would be better

$\sigma_4 = 3, 8, 2, 5, 6, 3, 8, 2, 5, 6, \sigma_1$ would be even better

$\boxed{\text{If } NPV(\sigma_1) > NPV(\sigma_2) \quad 3, 7, 4, 5, \ldots}$

Suggests that one should choose the same next state each time.

A policy $\pi : V \longrightarrow V$

says when in state i, move to $\pi(i)$ next.

$i_0, \pi(i_0), \pi^2(i_0), \ldots$

Problem: find optimal policy.

Fix π. Then for each $i \in V$,

$y_i :=$ N.P.V. $(i, \pi(i), \pi^2(i), \pi^3(i), \ldots)$

Find a policy that simultaneously minimises all y_i, $i \in V$.