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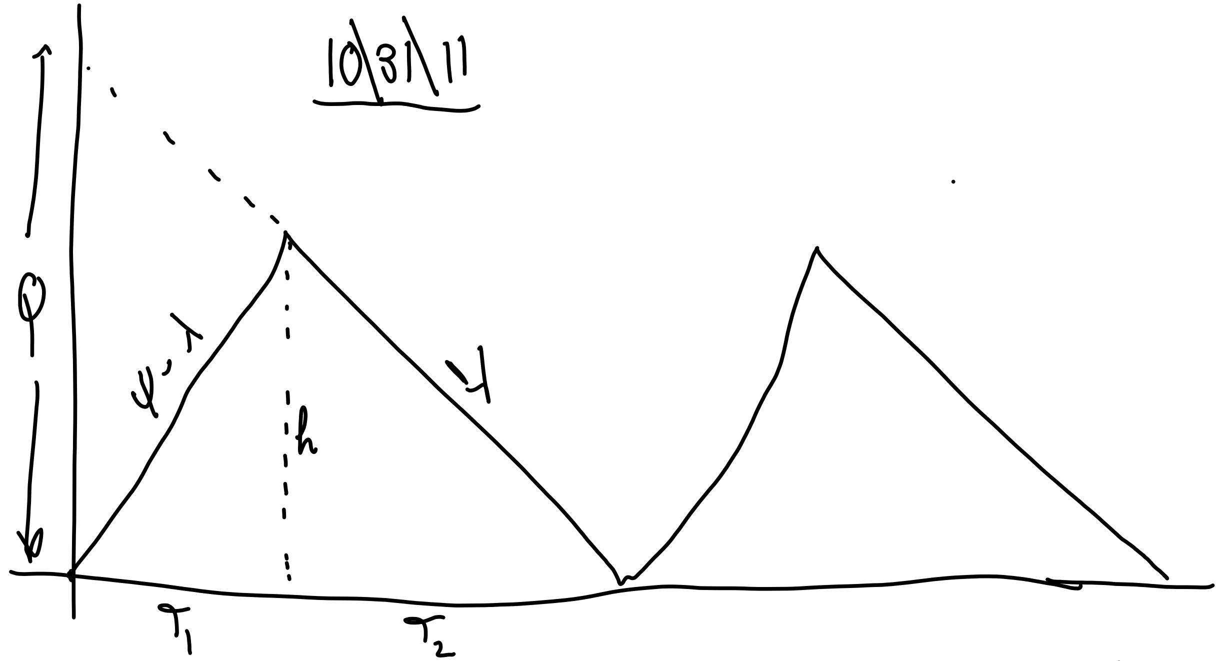



$$\text{Total Cost} = \underset{\text{Ordering}}{\frac{A\lambda}{Q}} + \underset{\text{Inventory}}{\frac{I h}{2}} = \frac{A\lambda}{Q} + \frac{I\left(1-\frac{\lambda}{\psi}\right)Q}{2}$$

$h = (\psi - \lambda)\, T_1 \qquad T_1 + T_2 = T = \dfrac{Q}{\lambda} \Rightarrow h = \left(1 - \dfrac{\lambda}{\psi}\right) Q$

$h = \lambda\, T_2$

Optimize w.r.t. $Q$.

$$Q^* = Q_W \left( \frac{\psi}{\psi - \lambda} \right)^{\frac{1}{2}}$$

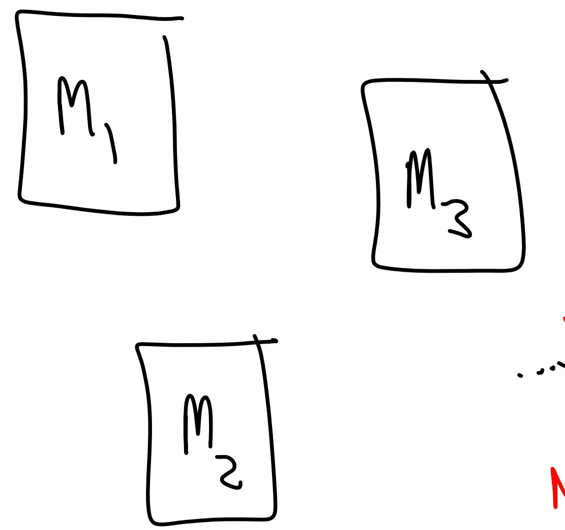
Wilson

$$K^* = K_W \left( \frac{\psi - \lambda}{\psi} \right)^{\frac{1}{2}}$$

# Job Shop Scheduling

Jobs

$J_1, J_2, \ldots, J_n$




Must work out the order in which to do each job on each machine.

NP-hard

A job has to processed on some of the machines. Want to minimise some quantity.

# Some simple solvable cases.

$$1 \,|\,|\, \sum w_j C_j$$

One machine

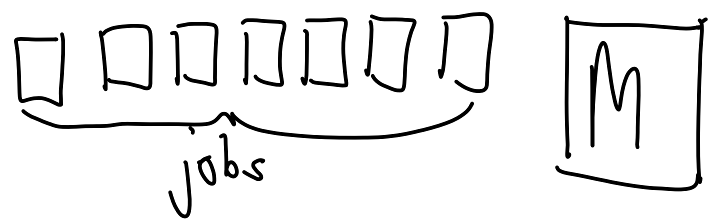
restrictions comments

weights

objective

$C_j$ = completion time of job $j$

$p_j$ = time to process job $j$



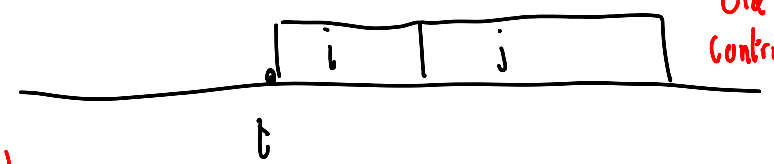

What is the best order?

$$\frac{w_1}{p_1} \geq \frac{w_2}{p_2} \geq \cdots$$

after re-numbering

## Proof

Suppose we have not used this ordering.

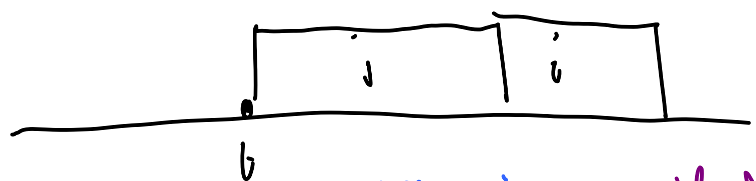



Old contribution: $w_i(t+p_i) + w_j(t+p_i+p_j)$

Interchanging will improve the objective

$$\frac{w_i}{p_i} < \frac{w_j}{p_j}$$

New contribution: $w_j(t+p_j) + w_i(t+p_i+p_j)$




Only $J_i$ & $J_j$ change their completion time.

Old - New = $-w_i p_j + w_j p_i > 0.$

$1 \| \; \| L_{max}$

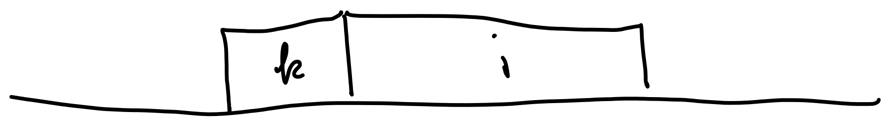
Lateness

Job $j$ has a due date $d_j$

$$L_j = (C_j - d_j)^+$$

Sort so that $d_1 \le d_2 \le \dots \le d_n$

Suppose we ordered so that




$d_j < d_k$

Interchange:

$$\text{Old} = \max\left(C_j^{old} - d_j,\; C_k^{old} - d_k\right) = C_j^{old} - d_j$$

Only $J_k$ & $J_j$ change their completion time.

$$\text{New} = \max\Big(\underbrace{C_j^{New} - d_j}_{< C_j^{old} - d_j},\; \underbrace{C_j^{old} - d_k}_{< C_j^{old} - d_j}\Big)$$