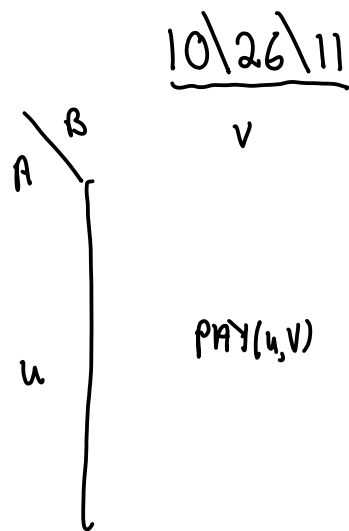




Game is stable iff

$$\max_u \text{ROWMIN}(u) = \min_v \text{COLMAX}(v)$$

Question: how to play the game when it isn't stable

We use randomization.

A chooses $p_1, p_2, \dots, p_m$ and chooses u with probability p_u
 B chooses $q_1, \dots, q_n$

$$\begin{matrix} & \underline{q} \\ \underline{p} & \left[\begin{matrix} \text{PAY}(\underline{p}, \underline{q}) \end{matrix} \right] \end{matrix}$$

$$\text{PAY}(\underline{p}, \underline{q}) = E(\text{winnings for A})$$

$$= \sum_{i=1}^m \sum_{j=1}^n a_{ij} p_i q_j$$

Can we always find $\underline{p}, \underline{q}$

so that this game is

stable?

We compute P_A, P_B and see
if $P_A = P_B$

$$P_A = \max_{\underline{P}} \min_{\underline{Q}} \sum_{i=1}^m \sum_{j=1}^n a_{ij} p_i q_j$$

$$= \max_{\underline{P}} \min_{\underline{Q}} \sum_{j=1}^n \left(\sum_{i=1}^m a_{ij} p_i \right) q_j$$

$$= \max_{\underline{P}} \min_{\underline{Q}} \sum_{j=1}^n c_j(\underline{P}) q_j :$$

$$c_j(\underline{P}) = \sum_{i=1}^m a_{ij} p_i$$

$$= \max_{\underline{P}} \min_{j=1, \dots, n} c_j(\underline{P})$$

$$P_A = \max_p \underbrace{\min_{j=1, \dots, n} c_j(p)}_{\mathcal{Z}}$$

$$= \text{maximum } \mathcal{Z}$$

$$\mathcal{Z} \leq \sum_{i=1}^m a_{ij} p_i \quad j=1, 2, \dots, n$$

$$p_1 + p_2 + \dots + p_m = 1$$

$$p_1, p_2, \dots, p_m \geq 0$$

$$P_B = \min_{\underline{q}} \max_{\underline{p}} \sum_{i=1}^m \sum_{j=1}^n a_{ij} p_i q_j$$

$$= \min_{\underline{q}} \max_{\underline{p}} \sum_{i=1}^m \left(\sum_{j=1}^n a_{ij} q_j \right) p_i$$

$$= \min_{\underline{q}} \max_{\underline{p}} \sum_{i=1}^m \underline{r_i(q)} p_i :$$

$$r_i(q) = \sum_{j=1}^n a_{ij} q_j$$

$$= \min_{\underline{q}} \min_{i=1,2,\dots,m} r_i(q)$$

$$\rho_B = \min_{\underline{q}} \underbrace{\max_{i=1,2,\dots,m} r_i(\underline{q})}_{\eta}$$

= minimum η

$$\eta \geq \sum_{j=1}^n a_{ij} q_j \quad i=1,2,\dots,m$$

$$q_1 + q_2 + \dots + q_n = 1$$

$$q_1, \dots, q_n \geq 0$$

maximum

z

$$z \leq \sum_{i=1}^m a_{ij} p_i$$

$$j=1, 2, \dots, n$$

$$p_1 + p_2 + \dots + p_m = 1$$

$$p_1, p_2, \dots, p_m \geq 0$$

DUAL
PAIR

minimum

w

$$w \geq \sum_{j=1}^n a_{ij} q_j$$

$$i=1, 2, \dots, m$$

$$q_1 + q_2 + \dots + q_n = 1$$

$$q_1, \dots, q_n \geq 0$$