

$$\underline{10/14/11}$$

Gomory Cuts:

Pure Integer Program:

$$\min x_0 = \underline{c}^T \underline{x}$$

$$A \underline{x} = \underline{b}$$

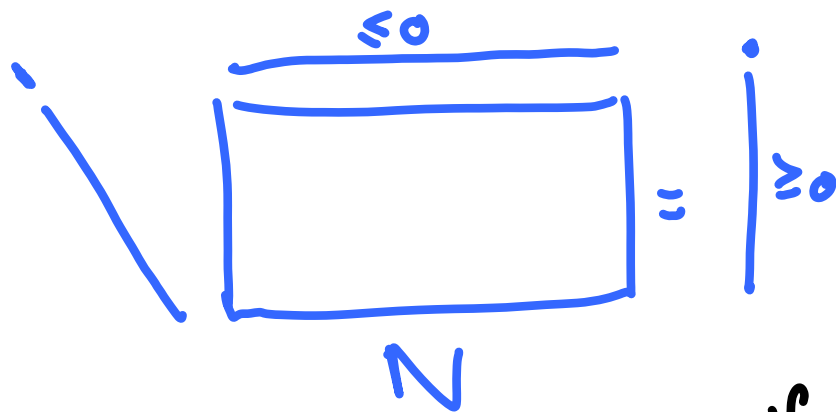
$$x_0, \underline{x} \geq 0 \quad \& \text{ integer.}$$

Suppose we solve the linear relaxation.

End up with a Simplex tableau

$$x_i + \sum_{j \in N} b_{ij} x_j = b_{i,0}$$

for all basic
variable $x_i \in \{0\} \cup I$.



indices of
the basic
variables

if $b_{i,0}$ is integer for all i then
this solves IP as well.

We suppose not i.e. there exist i such that $b_{i,0} \notin \mathbb{Z}$

$$x_i + \sum_{j \in N} b_{ij} x_j = b_{i,0}$$

We will find a constraint that is satisfied by all non-negative integral solutions to but is not satisfied by putting

$$x_i = b_{i,0}, \quad x_j = 0, \quad j \in N.$$

In general consider the equation

$$(*) \quad a_1 x_1 + a_2 x_2 + \dots + a_n x_n = b \quad a_1, \dots, a_n, b \in \mathbb{R}.$$

Write $a_i = \lfloor a_i \rfloor + f_i$ where $0 \leq f_i < 1$

$$6\frac{1}{2} = 6 + \frac{1}{2} \quad \text{and} \quad -7\frac{1}{4} = -8 + \frac{3}{4}$$

$$b = \lfloor b \rfloor + f$$

Substituting, we get

$$\sum_{j=1}^n f_j x_j - f = \lfloor b \rfloor - \sum_{j=1}^n \lfloor a_j \rfloor x_j$$

$$(*)*) \quad \sum_{j=1}^n f_j x_j - f = [b] - \sum_{j=1}^n [a_j] x_j$$

This holds for all solutions to $(*)$.

Suppose in addition that $x_1, x_2, \dots, x_n \in \mathbb{Z}$
 then RHS of $(**)$ is an integer.

So $\sum_{j=1}^n f_j x_j - f$ is also an integer.

$\sum_{j=1}^n f_j x_j - f$ is also an integer.

Suppose now that $x_1, \dots, x_n$ are also ≥ 0 .

Then

$$\sum_{j=1}^n f_j x_j - f \geq \cancel{-f} \geq 0 \text{ as } f < 1.$$

All non-negative integer solutions to (*)

Satisfy $\sum_{j=1}^n f_j x_j \geq f$

Apply this to

$$x_i + \sum_{j \in N} b_{ij} x_j = b_{i,0}$$

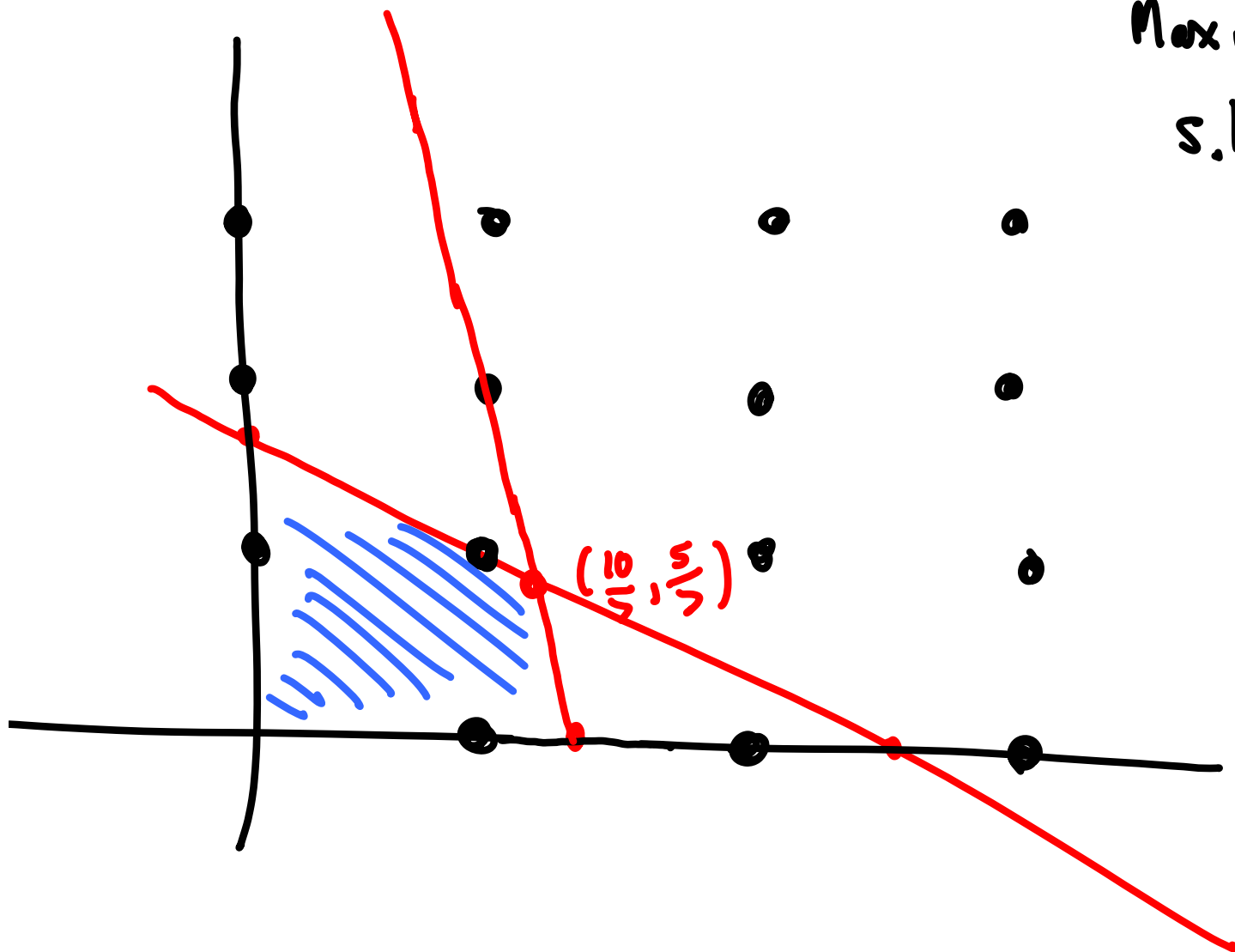
$$\underline{1} = 1 + 0$$

$$0x_i + \sum_{j \in N} f_j x_j \geq f$$

LP optimum does not satisfy this, as $0c_j = 0, j \in N$
and $f > 0$.

So we have a cut

Maximize $3x_1 + 4x_2$
 s.t. $2x_1 + 3x_2 \leq 5$
 $4x_1 - x_2 \leq 5$
 $x_1, x_2 \geq 0$
 & integer.



<u>R.V.</u>	x_1	x_2	x_3	x_4	R
x_0	-3	-4			0
x_3	2	(3)	1		5
x_4	4	-1		1	5

<u>R.V.</u>	x_1	x_2	x_3	x_4	P
x_0	$-\frac{1}{3}$		$\frac{1}{4}$		$\frac{2}{10}$
x_2	$\frac{2}{3}$	1	$\frac{1}{3}$		$\frac{2}{10}$
x_4	$\frac{1}{3}$		$\frac{1}{6}$	1	$\frac{2}{10}$

LP optimum

<u>R.V.</u>	x_1	x_2	x_3	x_4	R
x_0	$-\frac{1}{3}$		$\frac{19}{14}$	$\frac{1}{14}$	$\frac{5}{10}$
x_2		1	$\frac{2}{7}$	$\frac{1}{7}$	$\frac{2}{15}$
x_1	1		$\frac{1}{14}$	$\frac{3}{14}$	$\frac{2}{15}$

