10\12\11

Example

n projects that could be undertaken

If activated project j will yield c_j in profit.

There are m periods and at most k projects can start.

Project j would require a_{ij} dollars in period i.

There are b_i dollars available in period i.

Which projects should be undertaken to maximise profit?

Maximise profit $= c_1x_1 + c_2x_2 + \cdots + c_nx_n$

Subject to

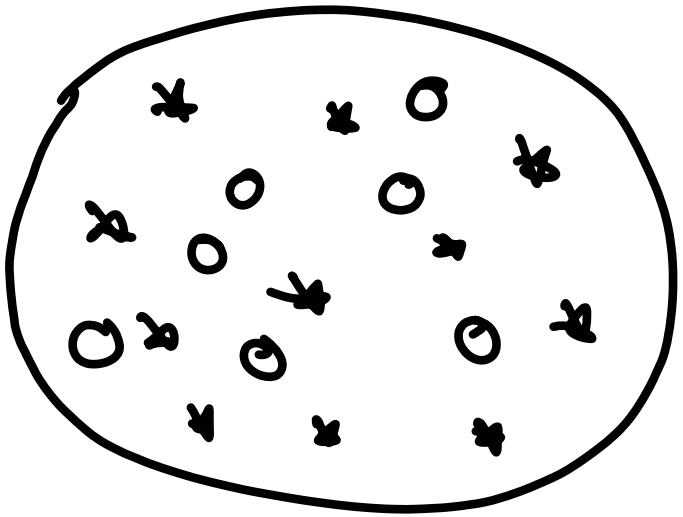
$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i, \quad i = 1, 2, \cdots, m$$

$$x_1 + x_2 + \cdots + x_n \leq k$$

$x_j = 0$ or 1 $\quad (0 \leq x_j \leq 1$ and integer$)$

$x_j = 1$ iff project j goes ahead.

Plant Location



Customer *

Possible site o
of plant

f_i = cost of a plant at site i

c_{ij} = cost of supplying j from i.

Minimise cost of building plants and shipping goods.

Minimise $\sum_{i=1}^{m} f_i y_i + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$

Building Cost + Shipping Cost

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad j = 1, 2, \cdots, n$$

$$x_{ij} \leq y_i \qquad \begin{matrix} i = 1, 2, \cdots, m \\ j = 1, 2, \cdots n \end{matrix}$$

$y_i = 0$ or 1 $\quad$ $y_i = 1$ iff build plant i.

$x_{ij} = 0$ or 1 $\quad$ $x_{ij} = 1$ iff i supplies j

Can relax this to $0 \leq x_{ij} \leq 1$

Set Covering Problem

Given sets $S_1, S_2, \ldots, S_n \subseteq S = \{1, 2, \ldots, m\}$

Costs $c_1, c_2, \ldots, c_n$

$I \subseteq \{1, 2, \ldots, n\}$ is a cover if $\bigcup_{i \in I} S_i = S$

$$c(I) = \sum_{i \in I} c_i$$

Problem: Find cover I that minimises $c(I)$

Minimize $\sum_{j=1}^{n} c_j x_j$

$a_{ij} = \begin{cases} 1 & i \in S_j \\ 0 & \text{otherwise} \end{cases}$

$i \in \bigcup_{\{j : x_j = 1\}} S_j$ for $i = 1, 2, \ldots, m$: For each i there must be a j such $x_j = 1$ and $i \in S_j$: $\sum_{j=1}^{n} a_{ij} x_j \geq 1, \quad i = 1, 2, \ldots, m$

$x_j = 0$ or 1 $\qquad$ $x_j = 1$ iff $S_j \in$ cover

Airline

$S = \{$ flight legs $\}$

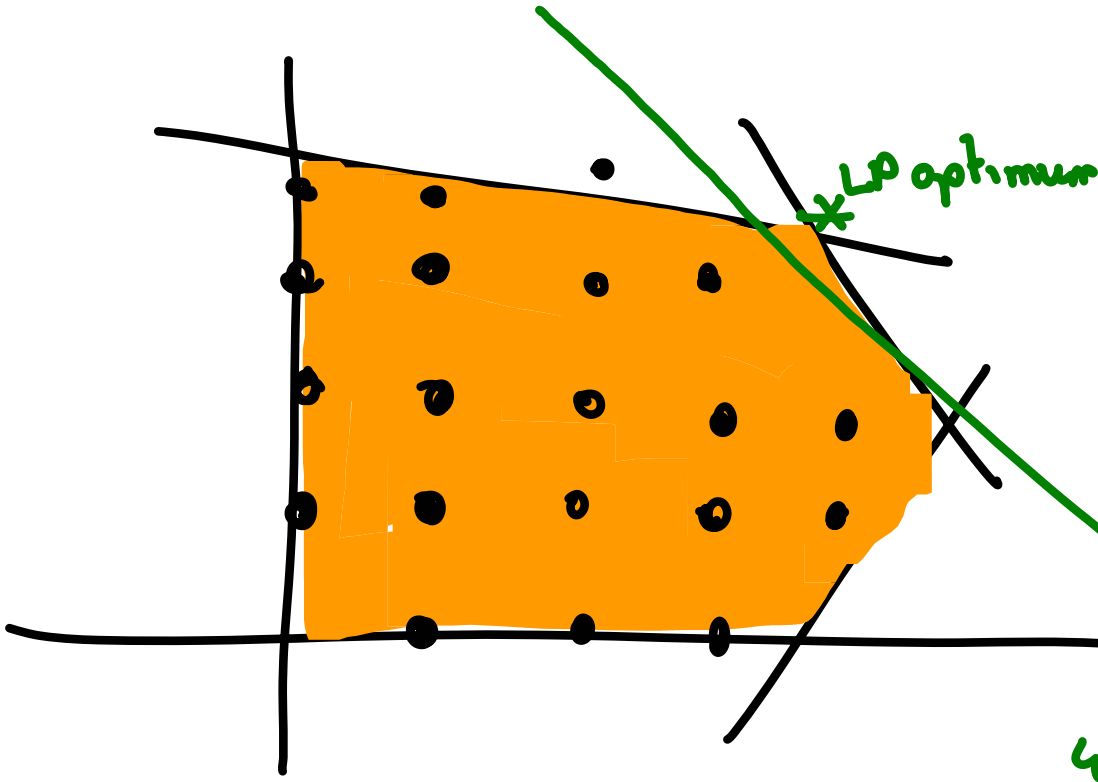
e.g. Pittsburgh → Newark ....

$S_i := \{$ set of legs that can be flown by a single crew $\}$

Pittsburgh → Newark, Newark → Toronto

⋮

Cutting Plane Algorithm



● – integer points

Solve LP

Lucky – optimum is a ● ✓

1: cuts off LP optimum

2: does not cut off any ●