Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 3: Due Friday November 5.

1. Find the optimal ordering strategy for the following inventory system. If you order an amount Q, it costs AQ^α for some $0 < \alpha < 1$ and the inventory cost is I per unit per period. The demand is λ units per period and no stock-outs are allowed.

   Solution: If we order Q units at a time then the total cost per period is
$$\frac{\lambda A Q^\alpha}{Q} + \frac{IQ}{2}.$$
   The optimal choice of Q is therefore $(2\lambda A(1-\alpha))^{1/(2-\alpha)}$.

2. Find an optimal inventory policy for the model with the following parameters: It is a generalisation of Models 2 and 3 of notes.

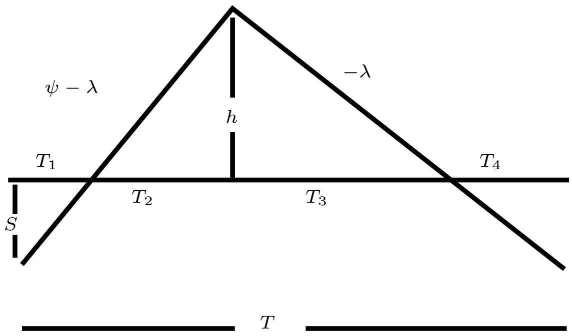
A	Cost of making an order.
λ	Demand per period for items.
ψ	Arrival rate of ordered items.
I	Inventory cost per item per period.
π	Penalty cost per item out of stock per period.

   (a) First draw a diagram showing the inventory level over time and various parameters.

   (b) Then identify the various costs per period.

   (c) Optimize total cost.

   Solution:



S and h will be our independent variables. Then

$$T_1 = \frac{S}{\psi - \lambda};\ T_2 = \frac{h}{\psi - \lambda};\ T_3 = \frac{h}{\lambda};\ T_4 = \frac{S}{\lambda}.$$

$$T = T_1 + T_2 + T_3 + T_4 = \frac{(S+h)\psi}{\lambda(\psi - \lambda)}.$$

Let K denote total cost. Then

$$\begin{aligned} K &= \frac{A}{T} + \frac{hI}{2} \cdot \frac{T_2 + T_3}{T} + \frac{\pi S}{2} \cdot \frac{T_1 + T_4}{T} \\ &= \frac{1}{S+h}\left(\frac{A\lambda(\psi - \lambda)}{\psi} + \frac{1}{2} I h^2 + \frac{1}{2}\pi S^2\right) \end{aligned}$$

Putting $\frac{\partial K}{\partial S} = \frac{\partial K}{\partial h} = 0$ we get

$$S^2 = \frac{2AI\lambda(\psi - \lambda)}{\pi\psi(I + \pi)} \text{ and } h^2 = \frac{2A\pi\lambda(\psi - \lambda)}{I\psi(I + \pi)}$$

3. Given that assigning person i to job i for $i = 1, 2, 3$ is optimal for the 3×3 problem associated with the first 3 rows and columns of the matrix below, find an optimal solution to the 4×4 problem:

$$\begin{bmatrix} 1 & 4 & 2 & 4 \\ 3 & 2 & 6 & 2 \\ 3 & 5 & 1 & 3 \\ 0 & 5 & 6 & 7 \end{bmatrix}$$