9/29/10

Introduce mixed strategies.

A chooses $p_1, p_2, \dots, p_m \geq 0$; $p_1 + \dots + p_m = 1$

B chooses $q_1, q_2, \dots q_n \geq 0$; $q_1 + \dots + q_n = 1$

A plays i with probability p_i

B plays j with probability q_j

Is there a stable choice of $\underline{p}, \underline{q}$?

Is $P_A = P_B$?

$$PAY(\underline{p}, \underline{q}) = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}\, p_i\, q_j$$

$$P_A = \max_{\underline{p}} \min_{\underline{q}} PAY(\underline{p}, \underline{q})$$

$$= \max_{\underline{p}} \left(\min_{\underline{q}} \sum_{j=1}^{n} \underbrace{\left(\sum_{i=1}^{m} a_{ij}\, p_i \right)}_{c_j(\underline{p})} q_j \right)$$

$$\min_{\underline{q}} \sum_{j=1}^{n} c_j(\underline{p})\, q_j = \min_{j} c_j(\underline{p})$$

$$P_A = \max_{\underline{p}} \underbrace{\min_{j} \sum_{i=1}^{m} a_{ij} p_i}_{\xi}$$

Maximise ξ

$$\xi - \sum_{i=1}^{m} a_{ij} p_i \leq 0, \quad j = 1, 2, \dots, n$$

$$\sum_{i=1}^{m} p_i = 1$$

$$p_i \geq 0, \quad i = 1, 2, \dots, m$$

$$P_B = \min_{\underline{q}} \max_{\underline{p}} \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \, p_i \, q_j$$

$$= \min_{\underline{q}} \left(\max_{\underline{p}} \sum_{i=1}^{m} \underbrace{\left(\sum_{j=1}^{n} a_{ij} \, q_j \right)}_{d_i(\underline{q})} p_i \right)$$

$$P_B = \min_{\underline{q}} \max_{\underline{p}} \sum_{i=1}^{m} d_i(\underline{q}) \, p_i$$

$$= \min_{\underline{q}} \max_{i} d_i(q)$$

$$P_B = \min_{\underline{q}} \max_i \sum_{j=1}^{n} a_{ij} q_j$$

$$= \min \; \eta$$

$$\eta - \sum_{j=1}^{n} a_{ij} q_j \geq 0 \quad i = 1, \dots, m$$

$$q_1 + \dots + q_n = 1$$

$$q_1, \dots, q_n \geq 0$$

Maximise ξ

$$\xi - \sum_{i=1}^{m} a_{ij} p_i \leq 0 , \quad j=1,2,\dots,n$$

$$\sum_{i=1}^{m} p_i = 1$$

$$p_i \geq 0, \quad i=1,2,\dots,m$$

min η

$$\eta - \sum_{j=1}^{n} a_{ij} q_j \geq 0 \quad i=1,\dots,m$$

$$q_1 + \dots + q_n = 1$$

$$q_1, \dots, q_n \geq 0$$

dual variables

$q_1, \dots, q_n, \eta$

≥ 0