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Knapsack Problem

$$f_r(w) = \max_{0 \le x \le \left\lfloor \frac{w}{w_r} \right\rfloor} \left[c_r x + f_{r-1}(w - w_r x) \right]$$

This idea could be used to solve

maximize $\quad c_1(x_1) + c_2(x_2) + \cdots + c_n(x_n)$

s.t. $\quad w_1(x_1) + w_2(x_2) + \cdots + w_n(x_n) \le W$

$\quad x_1, \cdots x_n \ge 0$ & integer

$$f_r(w) = \max_{\substack{w_r(x) \le w \\ x \ge 0}} \left[c_r(x) + f_{r+1}(w - w_r(x)) \right]$$

$$f_r(w) = \max_{0 \le x \le \lfloor \frac{w}{w_r} \rfloor} \left[c_r x + f_{r-1}(w - w_r x) \right]$$

arithmetic operation needed to use this is

$$O(nW^2)$$

Reduce this to $O(nW)$.

$$f_r(w) = \max \begin{cases} f_{r-1}(w) & x_r = 0 \\ c_r + f_r(w - w_r) & x_r \geq 1 \end{cases}$$

same meaning

$O(nW)$ time

$$\boxed{b = 0 \text{ or } 1}$$

Maximise $2x_1 + 4x_2 + 7x_3 + 10x_4$

Subject to $2x_1 + 3x_2 + 5x_3 + 7x_4 \le 14$

w	f_1	b_1	f_2	b_2	f_3	b_3	f_4	b_4
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
2	2	1	2	0	2	0	2	0
3	2	1	4	1	4	0	4	0
4	4	1	4	0/1	4	0	4	0
5	4	1	6	1	7	1	7	0
6	6	1	8	1	8	0	8	0
7	6	1	8	1	9	1	10	1
8	8	1	10	1	11	1	11	0
9	8	1	12	1	12	0	12	0/1
10	10	1	12	1	14	1	14	0/1
11	10	1	14	1	15	1	15	0
12	12	1	16	1	16	1	17	1
13	12	1	16	1	18	1	18	0/1
14	12	1	18	1	19	1	20	1

Suppose $W = 12$;

Knapsack & longest path