9\15\10

Suppose that π does not satisfy optimality criterion.

$$U = \{ i : \exists j \text{ with } y_i > c_{ij} + \alpha y_j \} \neq \emptyset$$

New policy $\hat{\pi}$,

$$\hat{\pi}(i) = \arg\min_j \, c_{ij} + \alpha y_j$$ j giving min

$\exists i : \hat{\pi}(i) \neq \pi(i)$

$$y_i \geq c_{i,\hat{\pi}(i)} + \alpha y_{\hat{\pi}(i)} \quad > \text{ for } i \in U$$

$$\hat{y}_i = c_{i,\hat{\pi}(i)} + \alpha \hat{y}_{\hat{\pi}(i)}$$

$$\underbrace{y_i - \hat{y}_i} \geq \alpha \left[y_{\hat{\pi}(i)} - \hat{y}_{\hat{\pi}(i)} \right]$$

$$\xi_i \geq \alpha \xi_{\hat{\pi}(i)} \geq \alpha^2 \xi_{\hat{\pi}^2(i)}$$

$$\geq \dots$$

$$\longrightarrow 0$$

$\xi_i > 0$ for $i \in U$

This could be solved by linear programming:

Need to solve

$$y_i = \min_j \left[c_{ij} + \alpha y_j \right] \quad i \in V.$$

put $\pi(i) =$ argmin j

Maximise $y_1 + y_2 + \cdots + y_n$

Constraints on y_i : $\quad y_i \leqslant c_{ij} + \alpha y_j \quad \forall i, j$