9\10\10

$c(x)$ = cost of producing x; H = max inventory

Demand d_n where

$$P[d_n = d] = p_{n,d}$$

π = penalty for not meeting demand

Minimise expected cost.

(1) choose production level in period n, before demand is known.

$$f_n(i) = \min_x \left[c(x) + \sum_d p_{n,d} \left[\pi(d-i-x)^+ + f_{n+1}\left[(x+i-d)^+\right] \right] \right]$$

really min H &

(1) choose production level in period n, after demand is known.

$$f_n(i) = \sum_d p_{n,d} \left(\min_x \left[c(x) + \pi(d-i-x)^+ + f_{n+1}(i+x-d)^+ \right] \right)$$

D.P. with ∞ horizon

Production costs

Policy 1

3 3 3 3 3 3 3 . . . .

↓ better average cost

Policy 2

2 2 2 2 2 2 2 . . .

↓ same average cost

Policy 3

1 3 1 3 1 3 1

Discounted Cash Flow

Net Present Worth

⋮

Evaluate your total expenditure in today's money – inflation

or

Evaluate in terms of how much you need to invest to generate the cost stream.

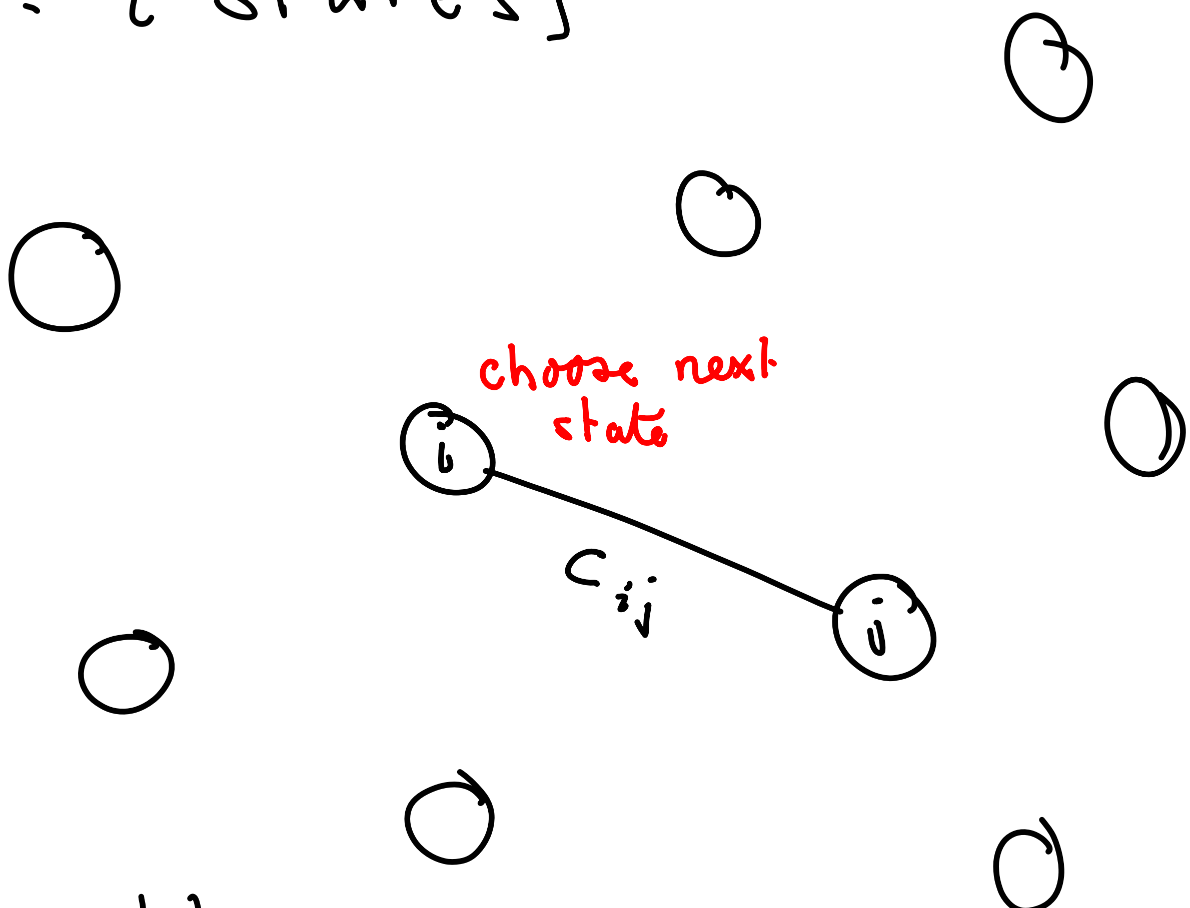
Parameter $\alpha \in (0,1)$

Value of $c_0, c_1, c_2, \ldots$

$\rightarrow \quad C_0 + \alpha C_1 + \alpha^2 C_2 + \cdots$

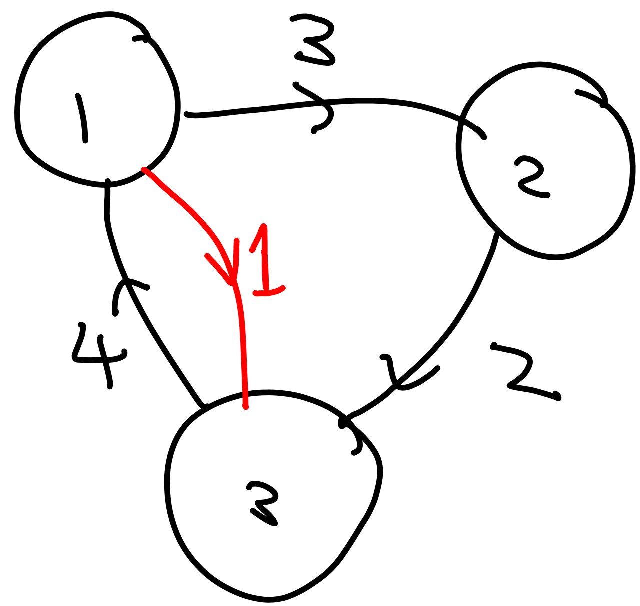
Simple Problem

$V = \{\text{states}\}$



More state → state indefinitely

α



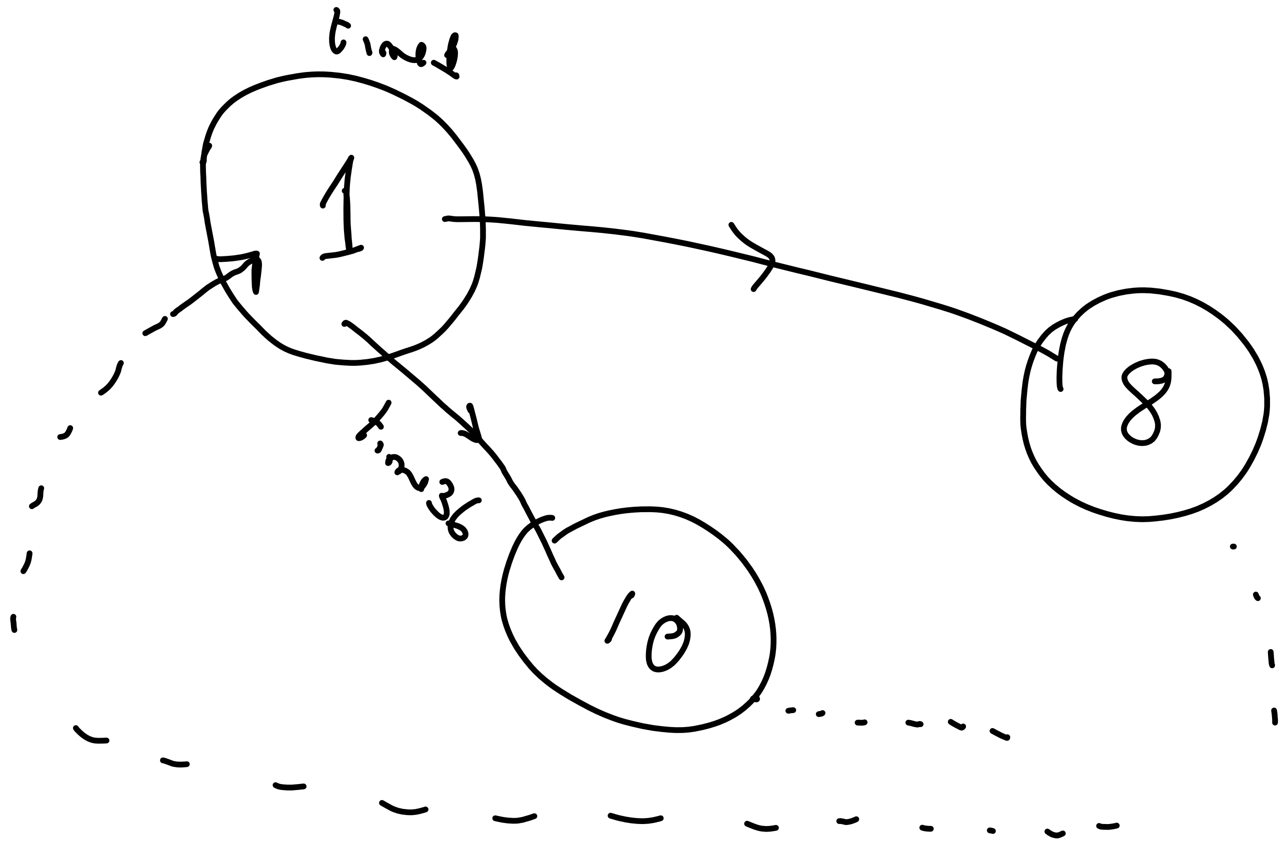
Start in 1

$$3 + 2\alpha + 4\alpha^2 + 3\alpha^3 + \ldots$$

There is discounted cost associated with any ∞ sequence of states

Choose sequence to minimise N.P.V.

Seems like we have an ∞ # of choices:



What you must do is choose a next state $\pi(i)$ for each $i \in V$. — policy π.

So, if $|V| = n$ then # choices is n^n.

Policy Iteration Algorithm:

Start with policy π_0

$\downarrow$ ← improve

π_1

$\downarrow$ ← improve

$\vdots$

π_N ← optimal.

$y_i = y_i(\pi) =$ NPV of π starting from i

We can choose π the simultaneously minimise <u>all</u> y_i, $i \in V$.

<u>Algorithm</u>

π_0

$\downarrow$ evaluate ie compute y_i's

$\downarrow$ check optimality

$\downarrow$ fail to be optimal

π_1