8\30\10

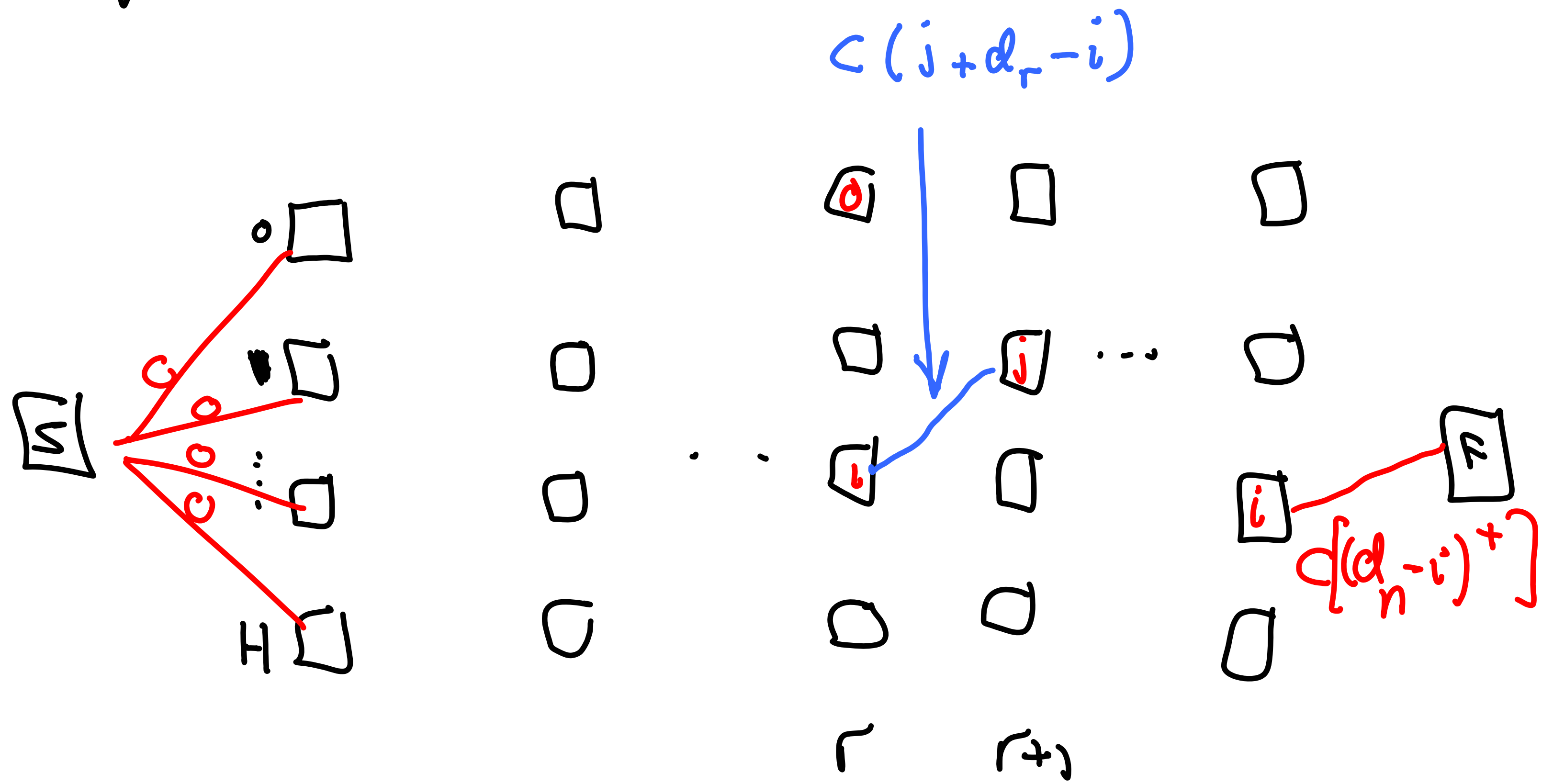
$n = 4;\ d = 3;\ H = 3;\ c(x) = 20x - x^2$

i	x_1	f_1	x_2	f_2	x_3	f_3	x_4	f_4
0	6	168	3 or 6	135	6	84	3	51
1	5	159	2	120	5	75	2	36
2	4	148	1	103	4	64	1	19
3	0 or 3	135	0	84	3	51	0	0

Suppose that we start with 1 in stock

$x_1 = 5,\ x_2 = 0,\ x_3 = 6,\ x_4 = 0.$ [Solution]

Equivalent mountain range problem.



$\boxed{i}$ start period r with i in stock

$$\xi^+ = \max\{0, \xi\}$$

Complicate the problems

(1) Add holding cost: $h(i_{average})$

average inventory in a period

$$f_r(i) = \min_x \left\{ c(x) + h\left(i + \frac{x-d_r}{2}\right) + f(i+x-d_r) \right\}$$

(ii) Allow items to be back-ordered up to maximum of B items.

Penalty π per item per period

Allow i to be negative

$$f_t(i) = \min_{x} \left\{ c(x) + \pi(-i)^+ + h\left(\frac{i^+ + (i+x-d_t)^+}{2}\right) + f_{t+1}(i+x-d_t) \right\}$$

$x \geq 0$

$i + x - d_t \in [-B, H]$

(vii) <u>Smoothing cost</u>. [Ignore holding and backorder

Suppose production levels are $x_1, x_2, \dots x_n$.

Add a cost of

$$\sigma(|x_2-x_1|) + \sigma(|x_3-x_2|) + \dots$$

$$f_r(i,y) = \min_x \Big[c(x) + \sigma(|x-\overset{\substack{\text{previous level}\\ \downarrow}}{y}|) + f_{r+1}(i+x-d_r, x) \Big]$$

Increases computation:

Got to compute $f_r(i,y)$ $\forall i,y$ instead of $\forall i$.

Knapsack Problem

Given weights $w_1, w_2, \dots, w_n$
profits $c_1, c_2, \dots, c_n$

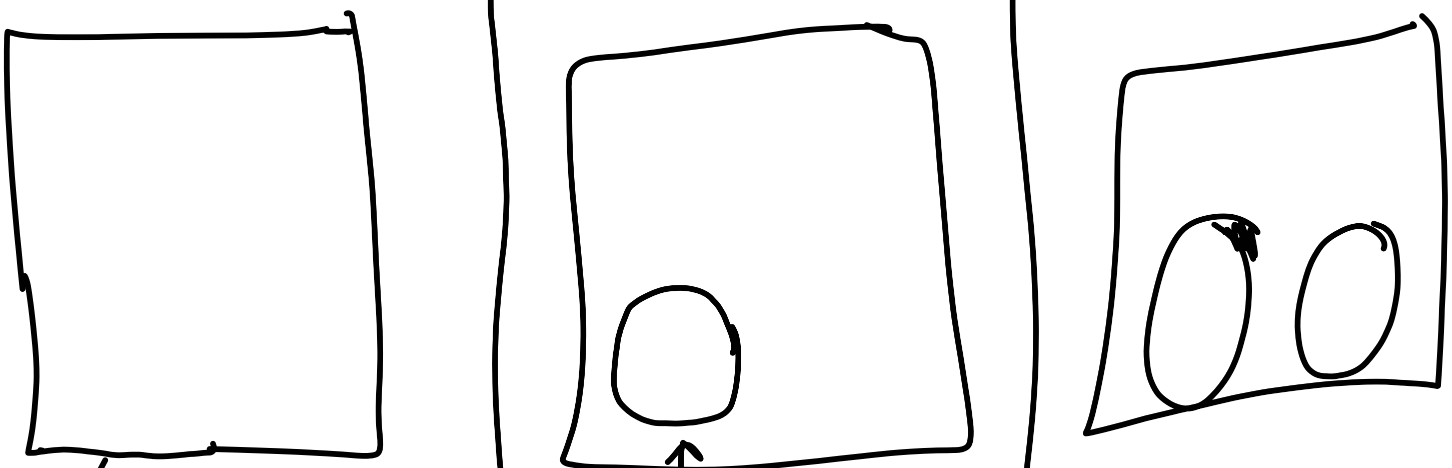
Problem

maximise $c_1 x_1 + \cdots + c_n x_n$

s.t. $w_1 x_1 + \cdots + w_n x_n \leq W$

$x_j \geq 0$ & integer.

Can solve $n-1$ item problems



$x_n = 0$ | $x_n = 1$ | $x_n = 2$ ...

$f_{n-1}(W)$ | $c_n + f_{n-1}(W - w_n)$ | $2c_n + f_{n-1}(W - 2w_n)$

$$f_n(W) = \max_{x} \left\{ c_n x + f_{n-1}(W - w_n x) \right\}$$

$$f_r(w) = \max \quad c_1x_1 + \cdots + c_rx_r$$

$$\text{s.t.} \quad w_1x_1 + \cdots + w_rx_r \leq w$$

$$x_1, \ldots x_r \geq 0 \text{ \& integer}$$

$$f_{r+1}(w) = \max_{0 \leq x \leq \lfloor \frac{w}{w_{r+1}} \rfloor} \left(c_{r+1}x + f_r(w - w_{r+1}x) \right),$$

$$f_0(w) = 0$$

Examp