10\8\10

Set Covering Problem

$S_1, S_2, \cdots S_n \subseteq S = \{1, 2, \cdots, m\}$

Costs are $c_1, c_2, \cdots, c_n > 0$

A cover is a set $I \subseteq \{1, 2, \cdots, n\}$

such

$$\bigcup_{i \in I} S_i = S$$

$$c(I) = \sum_{i \in I} c_i :$$

Problem: find cover I to minimize $c(I)$.

Airline Crew Scheduling

$S = \{\text{time-table}\} = \{\text{flights}\}$

S_i is a possible work load for a crew

$$x_j = \begin{cases} 1 & j \in I \\ 0 & j \notin I \end{cases}$$

$a_{ij} = 1_{i \in S_j}$

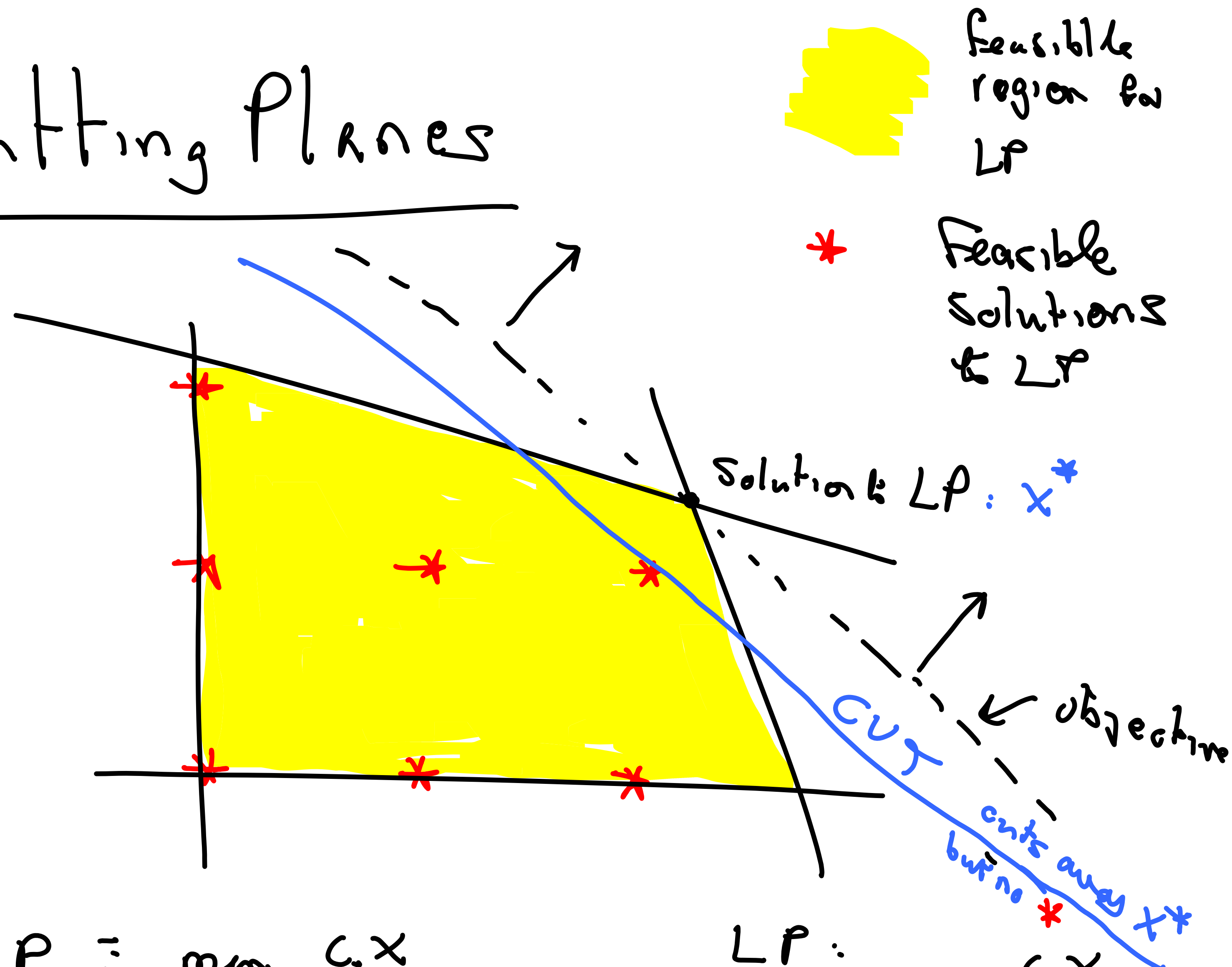
Objective:

$$\text{Cost} = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$$

$$a_{i1} x_1 + a_{i2} x_2 + \ldots + a_{in} x_n \geq 1$$

$x_j = 0$ or 1 $\quad j = 1, 2, \ldots n$

Cutting Planes



I.P. $\equiv$ min $c.x$

$$Ax \leq b$$
$$x \geq 0$$
$$x \text{ integer}$$

LP relaxation: min $c.x$

$$Ax \leq b$$
$$x \geq 0$$

Gomory Cuts [Pure problem: All variables must be integer]

Suppose we apply Simplex algorithm and find optimal solution is not integer.

Find a row of the tableau and a corresponding equation

$$x_i + \sum_{j \text{ non-basic}} b_{ij} x_j = b_{i0}$$

basic variable

currently zero

non-integer

In general, suppose we have an equation

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b$$

Write $\quad a_j = \lfloor a_j \rfloor + f_j \qquad\qquad \lfloor 6.3 \rfloor = 6$

$\qquad\quad b = \lfloor b \rfloor + f \qquad\qquad\qquad \lfloor -6.3 \rfloor = -7$

$$\sum_{j=1}^{n} \lfloor a_j \rfloor x_j = f - \sum_{j=1}^{n} f_j x_j \qquad (*)$$

Suppose that $x_1, x_2, \ldots, x_n$ are non-negative integers and that $(*)$ holds

LHS is integer $\Rightarrow$ RHS is integer & $\leq f < 1$

$\Rightarrow$ RHS is integer ≤ 0

$$x_i + \sum_{j \text{ non-basic}} b_{ij} x_j = b_{i0}$$

From this we get:

$$\sum_{j \text{ non-basic}} f_j x_j \geq f > 0$$

for integer solutions $\underline{x}$.

Current optimal LP solution has LHS = 0 & so does not satisfy