10\6\10

Integer Linear Programming

Linear programming where some variables have to be integer

$$\min \underline{c}^T \underline{x}$$
$$A \underline{x} = \underline{b}$$
$$\underline{x} \geq 0$$
$$x_i \in \{0, 1, 2, \ldots\}$$
$$\text{for } i \in I$$

Examples

(1) Capital Budgeting

n projects

m periods

project j has a return of c_j

requires a_{ij} dollars in period i

Money available in period i is b_i

Which projects maximise revenue?

$$X_j = \begin{cases} 0 & \text{do not do project} \\ 1 & \text{do project} \end{cases}$$

Formulate an I.P. to determine optimal X_j's.

Profit $= c_1 X_1 + c_2 X_2 + \cdots + c_n X_n$ [max]

Constraints

$$a_{i1} X_1 + a_{i2} X_2 + \cdots + a_{in} X_n \leq b_i \quad \forall i$$

$$X_j = 0 \text{ or } 1$$

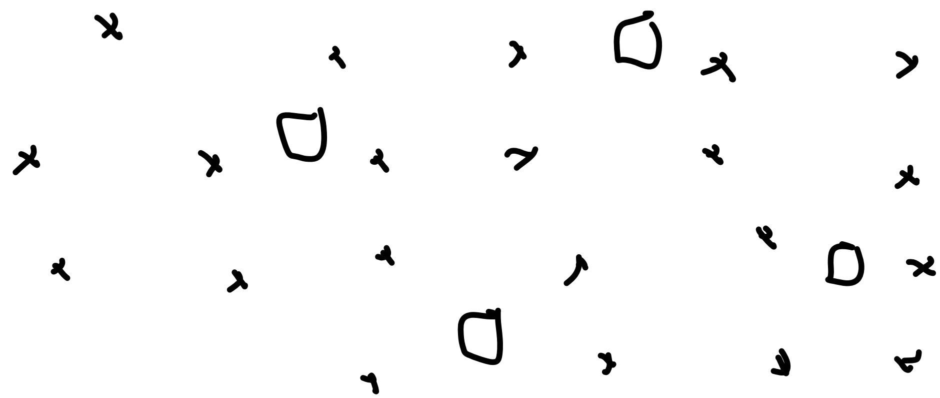
$0 \leq X_j \leq 1$, X_j integer

2)

Plant Location

× : customer

□ : possible location



n customers

m sites to build depots

c_{ij} = cost of supplying j from i.

f_i = cost of having depot at location i

Problem: determine which depots to build and how to supply customers.

$$y_i = \begin{cases} 0 & \text{not build } i \\ 1 & \text{build } i \end{cases} \qquad x_{ij} = \begin{cases} 0 & i \text{ not supplies } j \\ 1 & i \text{ supplies } j \end{cases}$$

Cost: $\underbrace{f_1 y_1 + \dots + f_m y_m}_{\text{construction}} + \underbrace{\sum_{i,j} c_{ij} x_{ij}}_{\text{supply cost}}$ (min)

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad \forall j$$

$$x_{ij} \leq y_i$$

$$x_{ij},\ y_i = 0 \text{ or } 1$$