

10/25/10

Inventory Control

Model 1

Wilson Lot-Size Model

Demand: constant and λ units per period

A: fixed cost of making an order.

C: unit cost per item (irrelevant)
(you have to pay

I: Inventory Cost
(λC per period
regardless

No "stock-out" allowed.



We choose T

$$\text{Total Cost} = \frac{A}{T} + \frac{IQ}{2}$$

ordering cost per period

inventory cost per period

$$Q = LT$$

Total cost $K = \frac{A\lambda}{Q} + \frac{IQ}{2}$

Now minimize w.r.t. Q

$$-\frac{A\lambda}{Q^2} + \frac{I}{2} = 0$$

$$Q = Q_w = \sqrt{\frac{2A\lambda}{I}}$$

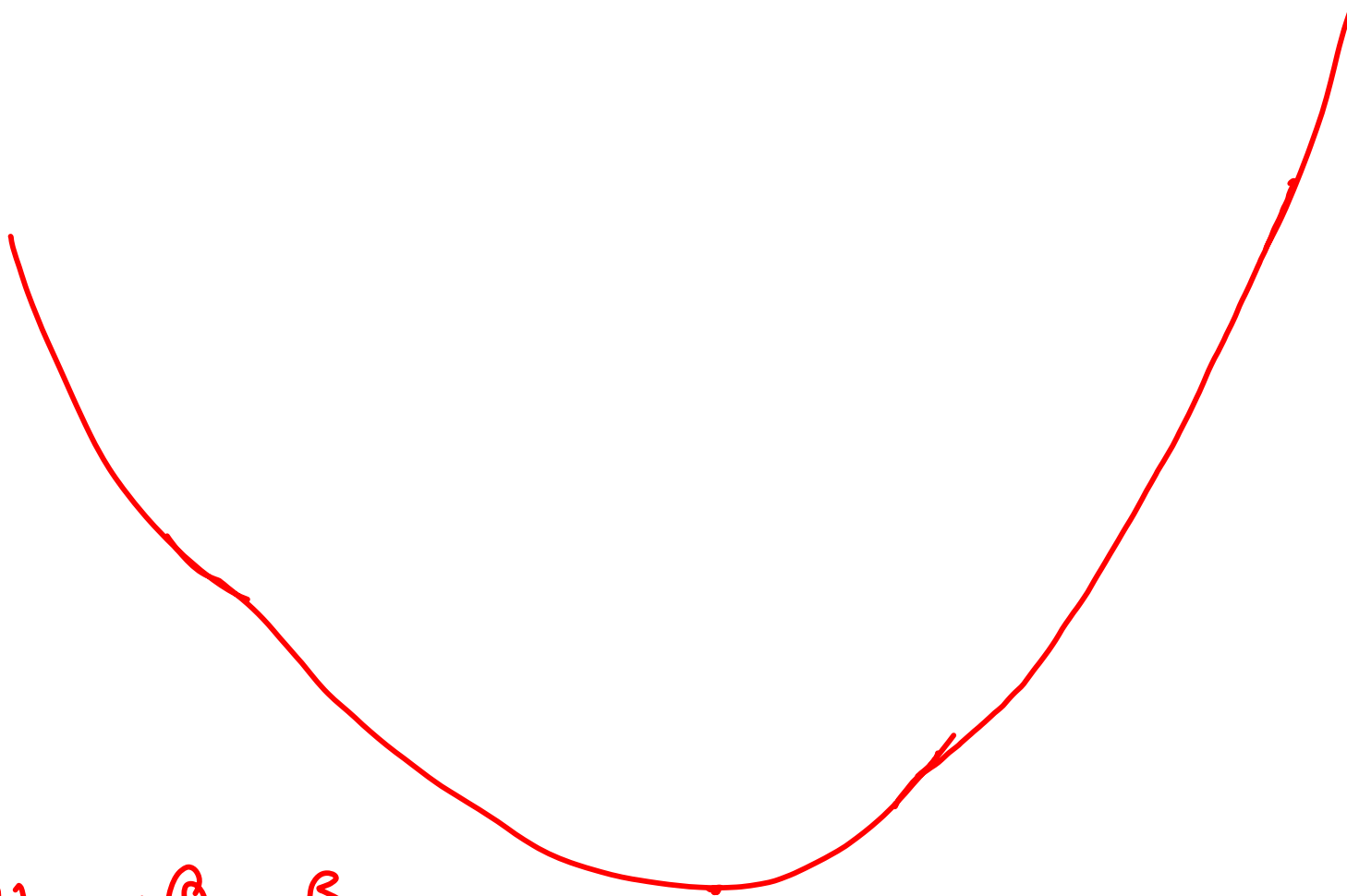
Wilson
Lot-Size
Formula

$$T_w = \sqrt{\frac{2A}{\lambda I}}$$

$$K_w = \sqrt{2\lambda AI}$$

$$\phi(Q) = \alpha Q + \frac{\beta}{Q}$$

$$\alpha, \beta > 0$$



Choose Q & S : (i) $T = \frac{Q}{\lambda}$ (ii) $T_2 = \frac{S}{\lambda}$
(iii) $T_1 = \frac{Q-S}{\lambda}$

A hand-drawn graph on a white background. The horizontal axis is a straight line. A sawtooth wave is drawn, starting from a point on the axis, rising linearly, and then dropping vertically. The slope of the rising part is labeled -1 . The period of the sawtooth wave is indicated by a bracket labeled 1 . A smooth, periodic curve is drawn, labeled Q with a bracket. The curve has a period of 2 , indicated by a bracket labeled 2 . The x-axis has two points marked with dots and labeled T_1 and T_2 . The sawtooth wave crosses the x-axis at T_1 and T_2 . The smooth curve Q crosses the x-axis at T_1 and T_2 .

$$\text{Total Cost} = \underbrace{\frac{A}{T}}_{\text{Ordering}} + \underbrace{I \times \frac{Q-S}{2} \times \frac{T_1}{T}}_{\text{Inventory}} + \underbrace{\pi \times \frac{S}{2} \times \frac{T_2}{T}}_{\text{Penalty}}$$

Putting it all together:

$$K = \frac{\lambda A}{Q} + \frac{I(Q-S)^2}{2Q} + \frac{\pi S^2}{2Q}$$

Put $\frac{\partial K}{\partial Q} = \frac{\partial K}{\partial S} = 0$

$$S = \sqrt{\frac{2\lambda A I}{\pi(\pi+I)}}$$

$$Q = Q_w \sqrt{\frac{\pi+I}{\pi}}$$

$$K = K_w \sqrt{\frac{\pi}{\pi+I}}$$