10\25\10

Inventory Control

Model 1

Wilson Lot-Size Model

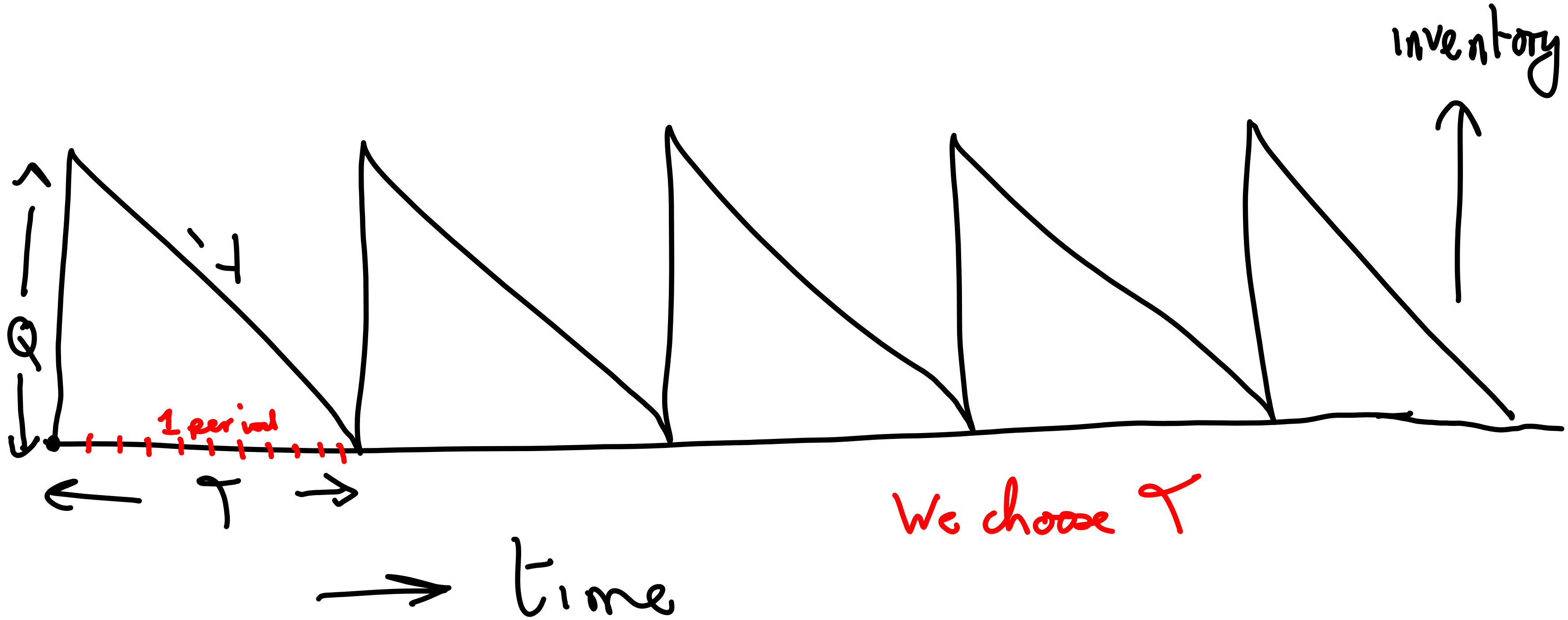
Demand: constant and λ units per period

A: fixed cost of making an order.

C: unit cost per item (irrelevant) (you have to pay (λC per period regardless

I: Inventory Cost

No "stock-out" allowed.



Total Cost: $\dfrac{A}{T} + \dfrac{IQ}{2}$ $\boxed{Q=\lambda T}$

$\dfrac{A}{T}$: ordering cost per period

$\dfrac{IQ}{2}$: inventory cost per period

Total cost $K = \frac{A\lambda}{Q} + \frac{IQ}{2}$

Now minimise w.r.t. Q

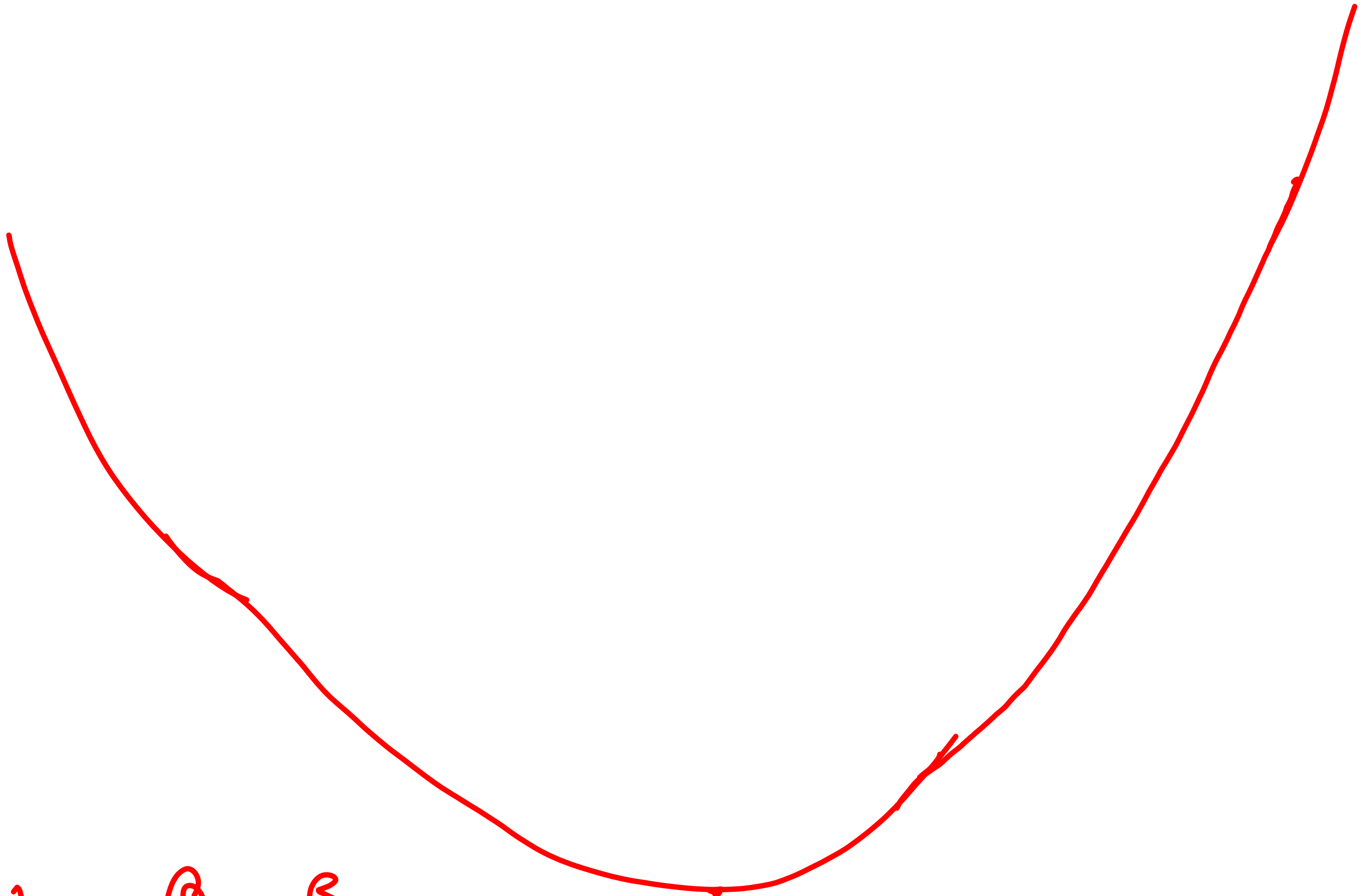
$$-\frac{A\lambda}{Q^2} + \frac{I}{2} = 0$$

$$Q = Q_W = \sqrt{\frac{2A\lambda}{I}}$$

Wilson Lot-Size Formula

$$T_W = \sqrt{\frac{2A}{\lambda I}}$$

$$K_W = \sqrt{2\lambda A I}$$



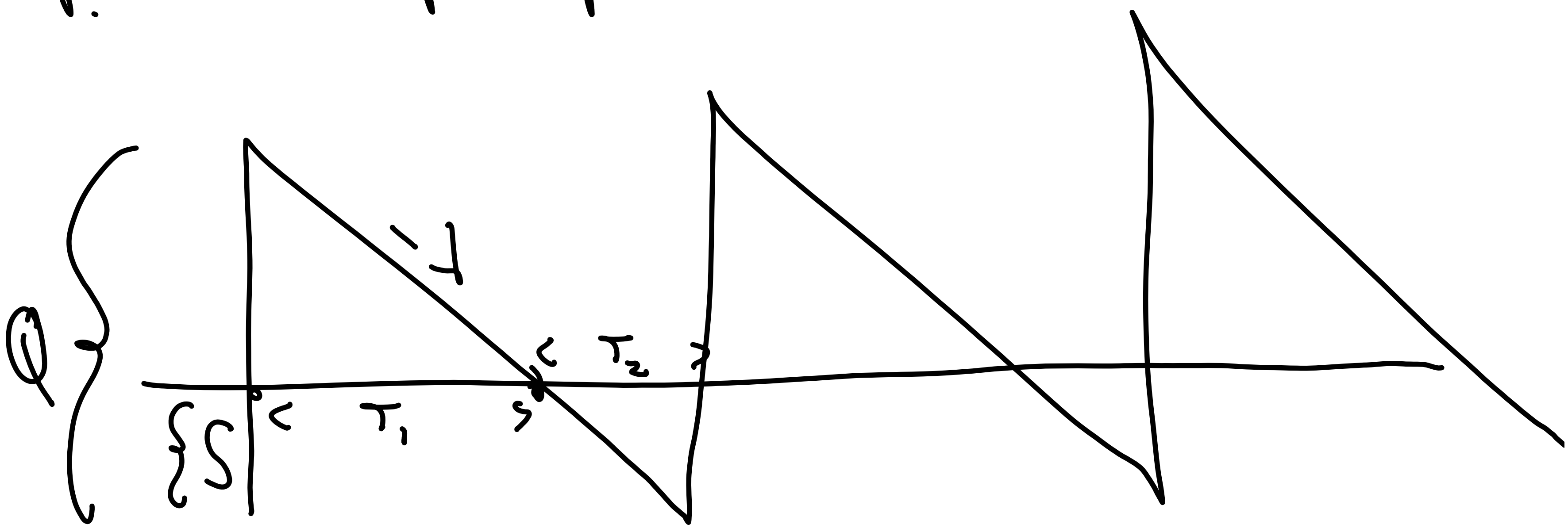
$$\phi(Q) = \alpha Q + \frac{\beta}{Q}$$

$$\alpha, \beta > 0$$

Model 2

Choose Q & S: (i) $T = \frac{Q}{\lambda}$ (ii) $T_1 = \frac{Q-S}{\lambda}$ (iii) $T_2 = \frac{S}{\lambda}$

Allow out of stock, but pay a penalty per unit per period



Total Cost: $\frac{A}{T} + I \times \frac{Q-S}{2} \times \frac{T_1}{T} + \pi \times \frac{S}{2} \times \frac{T_2}{T}$

Ordering Inventory Penalty

Putting it all together:

$$K = \frac{\lambda A}{Q} + \frac{I(Q-S)^2}{2Q} + \frac{\pi S^2}{2Q}$$

Put $\frac{\partial K}{\partial Q} = \frac{\partial K}{\partial S} = 0$

$$S = \sqrt{\frac{2\lambda A I}{\pi(\pi+I)}} \qquad Q = Q_w \sqrt{\frac{\pi+I}{\pi}}$$

$$K = K_w \sqrt{\frac{\pi}{\pi+I}}$$