Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 34: Due Wednesday November 18.

1. Find the optimal ordering strategy for the following inventory system. If you order an amount Q, it costs AQ^α for some $0 < \alpha < 1$ and the inventory cost is I per unit per period. The demand is λ units per period and no stock-outs are allowed.

   Solution: If we order Q units at a time then the total cost per period is
$$\frac{\lambda A Q^\alpha}{Q} + \frac{IQ}{2}.$$
   The optimal choice of Q is therefore $(2\lambda A(1-\alpha))^{1/(2-\alpha)}$.

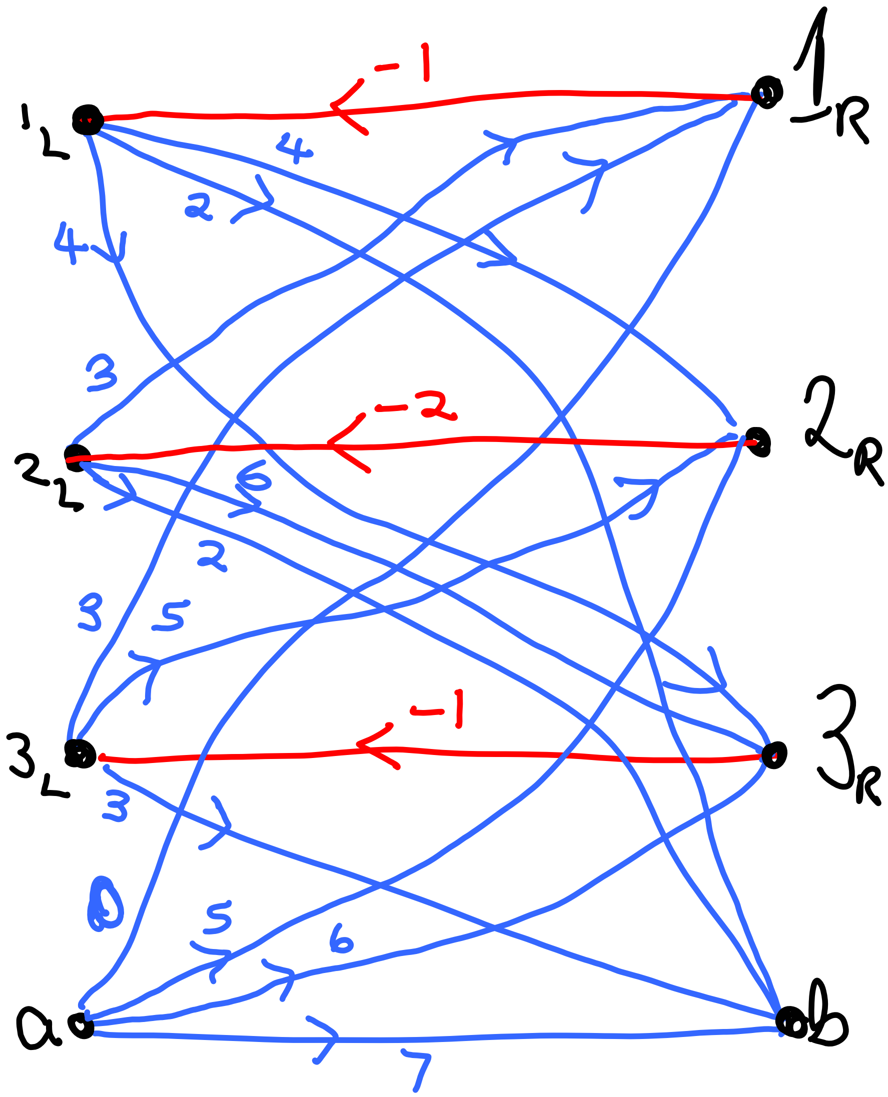
2. A cloth manufacturer sells rolls of cloth in n widths $\ell_1, \ell_2, \ldots, \ell_n$. Production is only in widths of width L. The manufacturer has to meet demand for d_j rolls of width ℓ_j and these must be cut from the larger rolls. For example if $\ell_1 = 7$ and $\ell_2 = 5$ and $L = 36$ then the manufacturer can cut 4 rolls of width 7 and 1 roll of width 5 from one large roll, leaving 3 feet of waste.

   The manufacturer wishes to meet demand and minimise total waste. Write an Integer Programming Formulation for this problem. The manufacturer will have to cut up several rolls in several different ways to solve this problem.

   Solution: Let Z_+ denote the set of non-negative integers and let $X = \{x \in Z^n : \ \ell_1 x_1 + \cdots + \ell_n x_n \leq L\}$. If the manufacturer cuts up m_x rolls with pattern x then the total waste is
$$L \sum_{x \in X} m_x - \sum_{j=1}^n d_j \ell_j$$
   and so minimising $\sum_{x\in X} m_x$ is equivalent to minimising waste. The problem is therefore
$$\begin{array}{lll} \text{Minimise} & \sum_{x\in X} m_x & \\ \text{Subject to} & \ell_j \sum_{x\in X} m_x x_j \geq d_j & j = 1, 2, \ldots, n \\ & m_x \in Z_+ & \forall x \in X. \end{array}$$

3. Given that assigning person i to job i for $i = 1, 2, 3$ is optimal for the 3×3 problem associated with the first 3 rows and columns of the matrix below, find an optimal solution to the 4×4 problem:

$$\begin{bmatrix} 1 & 4 & 2 & 4 \\ 3 & 2 & 6 & 2 \\ 3 & 5 & 1 & 3 \\ 0 & 5 & 6 & 7 \end{bmatrix}$$



Shortest path:

$a \to 1_R \to 1_L \to 3_R \to 3_L \to b$

optimal

1	4	(2)	4
3	(2)	6	2
3	5	1	(3)
(0)	5	6	7