Department of Mathematical Sciences CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 1: Due Monday September 7.

Q1 Solve the following knapsack problem:

maximise $4x_1 + 8x_2 + 13x_3$ subject to $3x_1 + 4x_2 + 5x_3 \leq 16$ $x_1, x_2, x_3 \geq 0$ and integer.

Q2 Consider a 2-D map with a horizontal river passing through its center. There are n cities on the southern bank with x-coordinates a(1)...a(n) and n cities on the northern bank with x-coordinates b(1)...b(n). You want to connect as many north-south pairs of cities as possible with bridges such that no two bridges cross. When connecting cities, you can only connect city i on the northern bank to city i on the southern bank. Construct a Dynamic Programming solution to this problem. (You can assume that $a(1) < a(2) < \cdots < a(n)$, but you **cannot** assume that $b(1) < b(2) < \cdots < b(n)$. If both sequences are increasing, then the problem is trivial).

Q3 Consider a row of n coins of values v(1)...v(n), where n is even. We play a game against an opponent by alternating turns. In each turn, a player selects either the first or last coin from the row, removes it from the row permanently, and receives the value of the coin. Construct a Dynamic Programming formulation that determines the maximum possible amount of money we can definitely win if we move first.