9\18\09

$\alpha = \frac{1}{2}$

$$\begin{bmatrix} 3 & 7 & 4 & 2 \\ 1 & 5 & 6 & 3 \\ 2 & 8 & 4 & 7 \\ 5 & 9 & 3 & 1 \end{bmatrix}$$

i	1	2	3	4
$\pi(i)$	4	1	1	4

$$y_1 = 2 + \tfrac{1}{2} y_4 = 3$$

$$y_2 = 1 + \tfrac{1}{2} y_1 = 5/2$$

$$y_3 = 2 + \tfrac{1}{2} y_1 = 7/2$$

$$y_4 = 1 + \tfrac{1}{2} y_4 = 2$$

$\alpha = \frac{1}{2}$ $\begin{bmatrix} 3 & 7 & 4 & 2 \\ 1 & 5 & 6 & 3 \\ 2 & 8 & 4 & 7 \\ 5 & 9 & 3 & 1 \end{bmatrix}$

y_1	y_2	y_3	y_4
3	5/2	7/2	2

$i = 1$:

$$3 + \tfrac{1}{2} y_1 = 9/2$$

$$7 + \tfrac{1}{2} y_2 = 33/4$$

$$4 + \tfrac{1}{2} y_3 = 23/4$$

$$2 + \tfrac{1}{2} y_4 = 3 \quad *$$

$i = 2$:

$$1 + \tfrac{1}{2} y_1 = 5/2 \quad *$$

$$5 + \tfrac{1}{2} y_2 =$$

$$6 + \tfrac{1}{2} y_3 =$$

$$3 + \tfrac{1}{2} y_4 =$$

$\alpha = \frac{1}{2}$ $\begin{bmatrix} 3 & 7 & 4 & 2 \\ 1 & 5 & 6 & 3 \\ 2 & 8 & 4 & 7 \\ 5 & 9 & 3 & 1 \end{bmatrix}$

y_1	y_2	y_3	y_4
3	5/2	7/2	2

OPTIMAZ

$i=3:$

$$2 + \frac{1}{2} y_1 = 7/2 \quad *$$

$$8 + \frac{1}{2} y_2 =$$

$$4 + \frac{1}{2} y_3 =$$

$$7 + \frac{1}{2} y_4 =$$

$i=4:$

$$5 + \frac{1}{2} y_1 =$$

$$9 + \frac{1}{2} y_2 =$$

$$3 + \frac{1}{2} y_3 =$$

$$1 + \frac{1}{2} y_4 = 2 \quad *$$

Theorem

π is optimal for all i

iff, $\forall i$

$$(*) \quad y_i = c_{i,\pi(i)} + \alpha y_{\pi(i)} = \min_j \left[c_{ij} + \alpha y_j \right]$$

stopping condition for algorithm

Proof

Suppose that (*) is true.

Let $\hat{\Pi}$ be any other policy and let $\hat{y}$ be its values.

Let $\xi_i = \hat{y}_i - y_i$

Want to show that $\xi_i \geq 0, \forall i$

$$\hat{y}_i = c_{i,\hat{\pi}(i)} + \alpha \hat{y}_{\hat{\pi}(i)}$$

$$- \quad y_i \leq c_{i,\hat{\pi}(i)} + \alpha y_{\hat{\pi}(i)}$$

optimality

$\circledast$

$$\xi_i \geq \alpha \xi_{\hat{\pi}(i)}$$

$$\geq \alpha^2 \xi_{\hat{\pi}^2(i)}$$

$$\vdots$$

$$\geq \alpha^k \xi_{\hat{\pi}^k(i)} \longrightarrow 0 .$$

Now suppose that (*)

fails:

Let $m(i)$ = index of min in (*) i.e.

$$c_{i,m(i)} + \alpha y_{m(i)} = \min_j \left[c_{ij} + \alpha y_j \right]$$

$m(i) = \pi(i)$ unless there is a <u>strictly</u>

better j.

$$U = \{ j : m(i) \neq \pi(i) \} \neq \emptyset$$

Now define $\tilde{\pi}(i) = m(i)$

and let $\tilde{y}$ be its values.

$$y_i \geq c_{i,\tilde{\pi}(i)} + \alpha\, y_{\tilde{\pi}(i)}$$

$$\tilde{y}_i = c_{i,\tilde{\pi}(i)} + \alpha\, \tilde{y}_{\tilde{\pi}(i)}$$

$$\eta_i = y_i - \tilde{y}_i .$$

Show $\eta_i \geq 0,\ \forall i$ and $\eta_i > 0,\ i \in U$

As before

$$\eta_i \geq \alpha \, \eta_{\tilde{\pi}(i)}$$

and $\quad \eta_i > \alpha \, \eta_{\tilde{\pi}(i)}, \quad i \in U$

We know

$$\eta_i \geq \alpha^k \eta_{\tilde{\pi}^k(i)} \quad \forall k$$

$$\vdots$$

$$\geq 0 \quad \text{in limit.}$$

& $\quad \eta_i > 0$, for $i \in U$

Introduce randomness

Decisions:

D_i

$d \in D_i$