9\14\09

Problems with an infinite time horizon.

Suppose there cost a_n of doing business in period n.

Possibilities for costs:

1) 3 3 3 3 3 3 . . .

2) 2 2 2 2 2 2

3) 1 3 1 3 1 3 . .

same total ∞ cost

1) worse than 2), 3)

3) better than 2) if there is inflation.

Net present value of sequence is

$$a_0 + a_1\alpha + a_2\alpha^2 + \cdots + a_n\alpha^n + \cdots$$

where $0 < \alpha < 1$.

Choose sequence to minimise

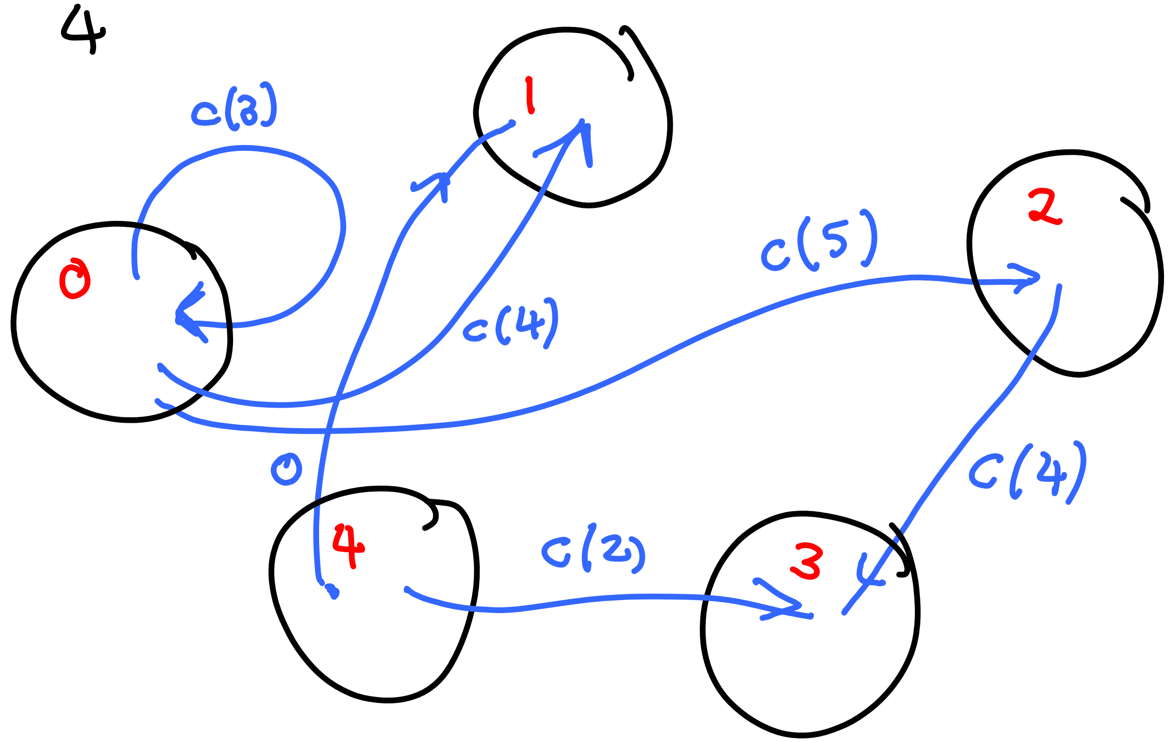
3) < 2) < 1)

Example

Production problem:

Demand = 3 every period

$H = 4$



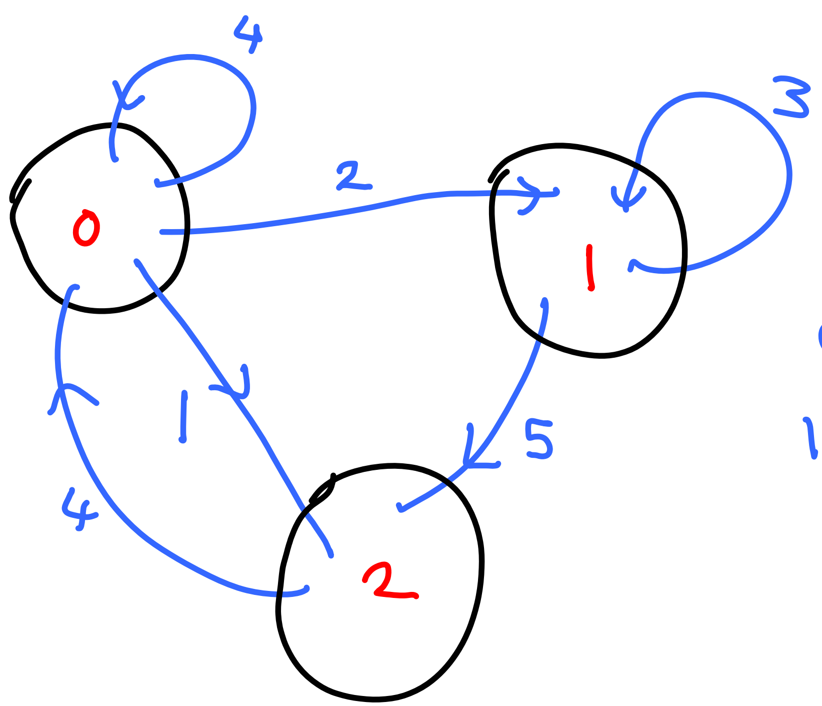
System is in one of n states, V.

$c_{i,j}$ = cost of going from state i to state j in one move.

α = discount factor

System runs for ever, going from state to state.

We need to determine the sequence of states that minimizes total disounted cost



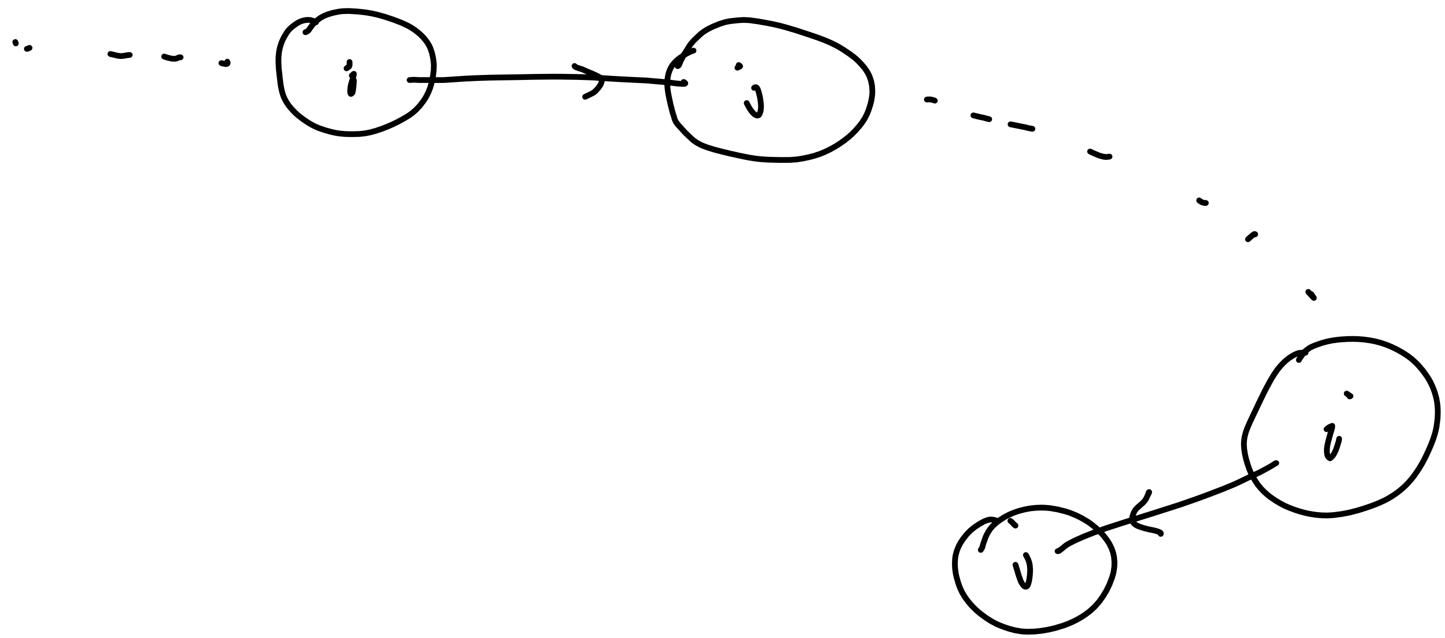
all other costs infinite

Cost of 0 1 2 0 1 2 0 1 2

$2 + 5\alpha + 4\alpha^2 + 2\alpha^3 + \dots$

Optimal Sequence:

here $\pi(i) = j$



Optimal policy should be

$$\pi : V \longrightarrow V$$

when in i go to $\pi(i)$

We want an algorithm to choose optimal π.

y_i = cost of running π, starting at i

$$= c_{i,\pi(i)} + \alpha c_{\pi(i),\pi^2(i)} + \alpha^2 c_{\pi^2(i),\pi^3(i)} + \dots$$

$$= c_{i,\pi(i)} + \alpha \Big[\underbrace{c_{\pi(i),\pi^2(i)} + \alpha c_{\pi^2(i),\pi^3(i)} + \dots}_{y_{\pi(i)}} \Big]$$

So to compute y_i, $i \in V$ we solve

$$y_i = c_{i,\pi(i)} + \alpha\, y_{\pi(i)}, \quad i \in V$$

$n = 4$; $\alpha = \frac{1}{2}$

i	1	2	3	4
$\pi(i)$	4	1	3	1

$$\begin{bmatrix} 3 & 4 & 2 & 6 \\ 5 & 3 & 1 & 5 \\ 2 & 1 & 5 & 4 \\ 6 & 3 & 4 & 2 \end{bmatrix}$$

$y_1 = 6 + \frac{1}{2} y_4 = 12$

$y_2 = 5 + \frac{1}{2} y_1 = 11$

$y_3 = 5 + \frac{1}{2} y_3 = 10$

$y_4 = 6 + \frac{1}{2} y_1 = 12$