8\31\09

Knapsack Problem

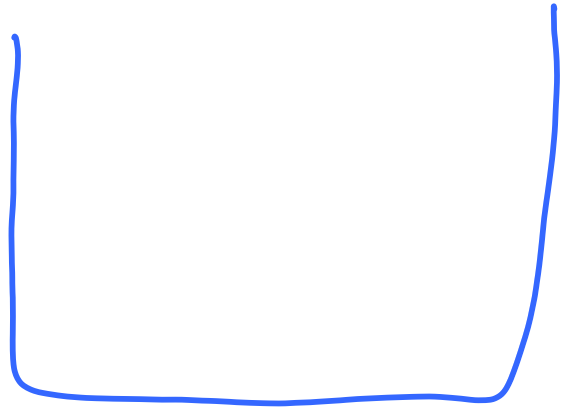
Maximize $c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$

Subject to $w_1 x_1 + w_2 x_2 + \cdots + w_n x_n \leq W$

$x_1, x_2, \ldots, x_n \geq 0 \in \text{Integer}$

$c_1, \ldots c_n, w_1, \ldots, w_n, W > 0$

Scout



Knapsack

n types of item

Each item have value c_j weight w_j

Pack x_1 of item 1

x_2 of item 2

$\vdots$

$$\text{Value} = c_1x_1 + \cdots + c_nx_n$$

$$\text{weight} = w_1x_1 + \cdots + w_nx_n$$

Scout can only carry W on trip

Plane

We can solve this in $O(nW)$ time.

One way to solve this is "branch & bound"

We use Dynamic Programming here.

Solution = sequence of decisions.

x_n = # of type n

$$\text{Max Value} = \max_{x_n} \left[c_n x_n + \text{Value}_{n-1}(W - w_n x_n) \right]$$

$$f_r(w) = \text{maximum} \quad c_1x_1 + \cdots + c_rx_r$$

$$w_1x_1 + \cdots + w_rx_r \leq w$$

$w = 0, 1, \ldots, W$

$$= \max_{\substack{x_r \geq 0 \\ x_r \leq \frac{w}{w_r}}} \left[c_rx_r + f_{r-1}(w - w_rx_r) \right]$$

$$f_0(w) = 0 \quad \text{or} \quad f_1(w) = c_1 \left\lfloor \frac{w}{w_1} \right\rfloor$$

Run in $O(nW^2)$ time.

Maximise $x_1 + 2x_2 + 4x_3$.

s.t. $2x_1 + 3x_2 + 7x_3 \leq 10$

$x_1, x_2, x_3 \geq 0$ & integer.

w	f_1	x_1	f_2	x_2	f_3	x_3
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	1	1	1	0	1	0
3	1	1	2	1	2	0
4	2	2	2	0 or 1	2	0
5	2	2	3	1	3	0
6	3	3	4	2	4	0
7	3	3	4	1 or 2	5	1
8	4	4	5	2	5	0 or 1
9	4	4	6	3	6	0 or 1
10	5	5	6	2 or 3	7	1