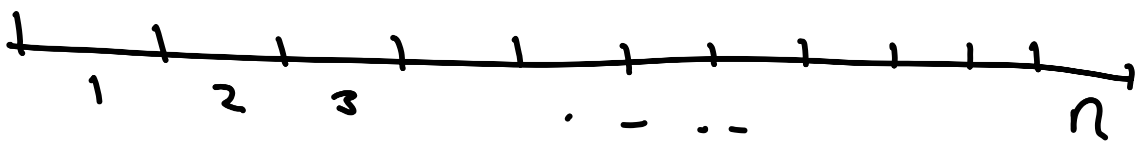
# 8/26/09

## Production Problem

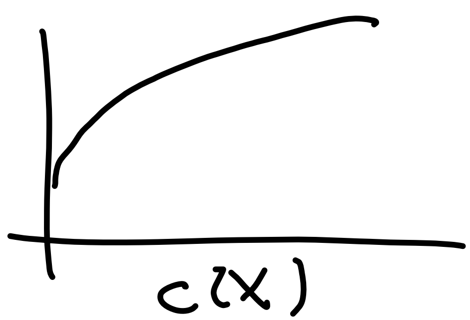



n periods

(I) Known demands $d_1, d_2, \dots, d_n$

Must supply $d_j$ items in period $j$

(II) cost $c(x)$ to make $x$.

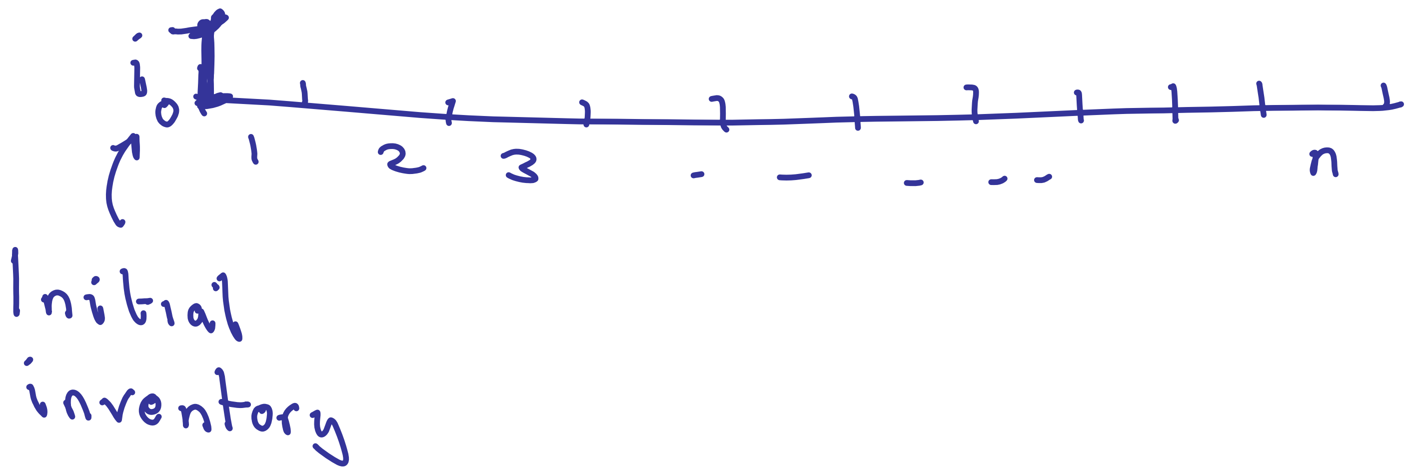



Answer: $c(d_1) + \dots + c(d_n)$

or

$c(d_1 + \dots + d_n)$

Assume you can store up to H items.

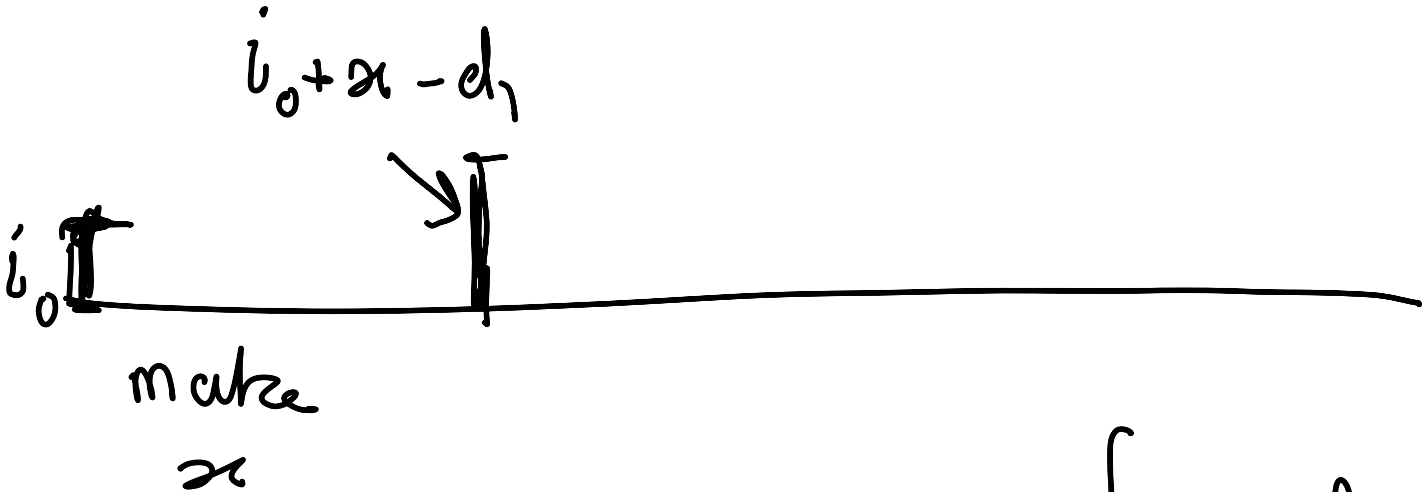



## Dynamic Programming:

Think: (i) Answer = sequence of decisions

(ii) Having made first decision, left with a smaller problem.

Decision $\equiv$ choose production quantity




$$f_1(i_0) = \min_x \left[ c(x) + f_2(i_0+x-d_1) \right]$$

min. cost from 1, 2, .. n starting with $i_0$

If I can compute this then I can compute

In general functional equation

$$f_r(i) = \min_{x} \left[ c(x) + f_{r+1}(i + x - d_r) \right]$$

min. cost periods $r, r+1, \ldots, n$ starting with $i$

$x \geq 0$

$i + x \geq d_r$

$i + x - d_r \leq H$

$$f_{n+1}(i) = 0$$

| | |
|---|---|
| 2 | 36 |
| 3 | 57 |
| 4 | 64 |
| 5 | 75 | 191 |
| 6 | 84 | 190 |
| 7 | 91 | 187 |
| 8 | 96 | 182 |
| 9 | 99 | 175 |

| $n = 4$ | $d_1$ | $d_2$ | $d_3$ | $d_4$ |
|---|---|---|---|---|
| $H = 4$ | 5 | 6 | 4 | 7 |

$$c(x) = x(20 - x)$$

| $i$ | $f_1(i)$ | $x_1(i)$ | $f_2(i)$ | $x_2(i)$ | $f_3(i)$ | $x_3(i)$ | $f_4(i)$ | $x_4(i)$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 266 | 5 | 191 | 10 | 147 | 8 | 91 | 7 |
| 1 | 255 | 4 | 190 | 9 | 142 | 3 or 7 | 84 | 6 |
| 2 | 242 | 3 | 187 | 8 | 127 | 2 | 75 | 5 |
| 3 | 227 | 2 | 182 | 7 | 110 | 1 | 64 | 4 |
| 4 | 210 | 1 | 175 | 6 | 91 | 0 | 51 | 3 |