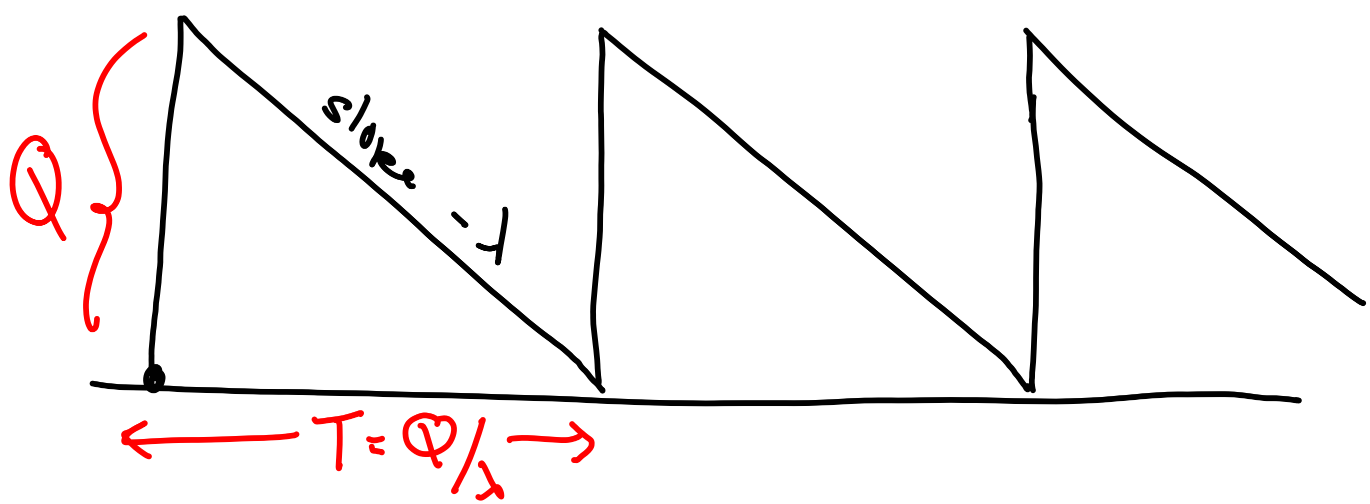
11\06\09

Inventory Control

Model 1

Demand = λ units per period

Inventory:



Costs: Order Cost A; I = inventory carrying per unit per period

Cost per period =

$\frac{A}{T} = \frac{A\lambda}{Q}$ $\qquad\qquad$ $\frac{IQ}{2}$

Order Cost + Inventory Cost

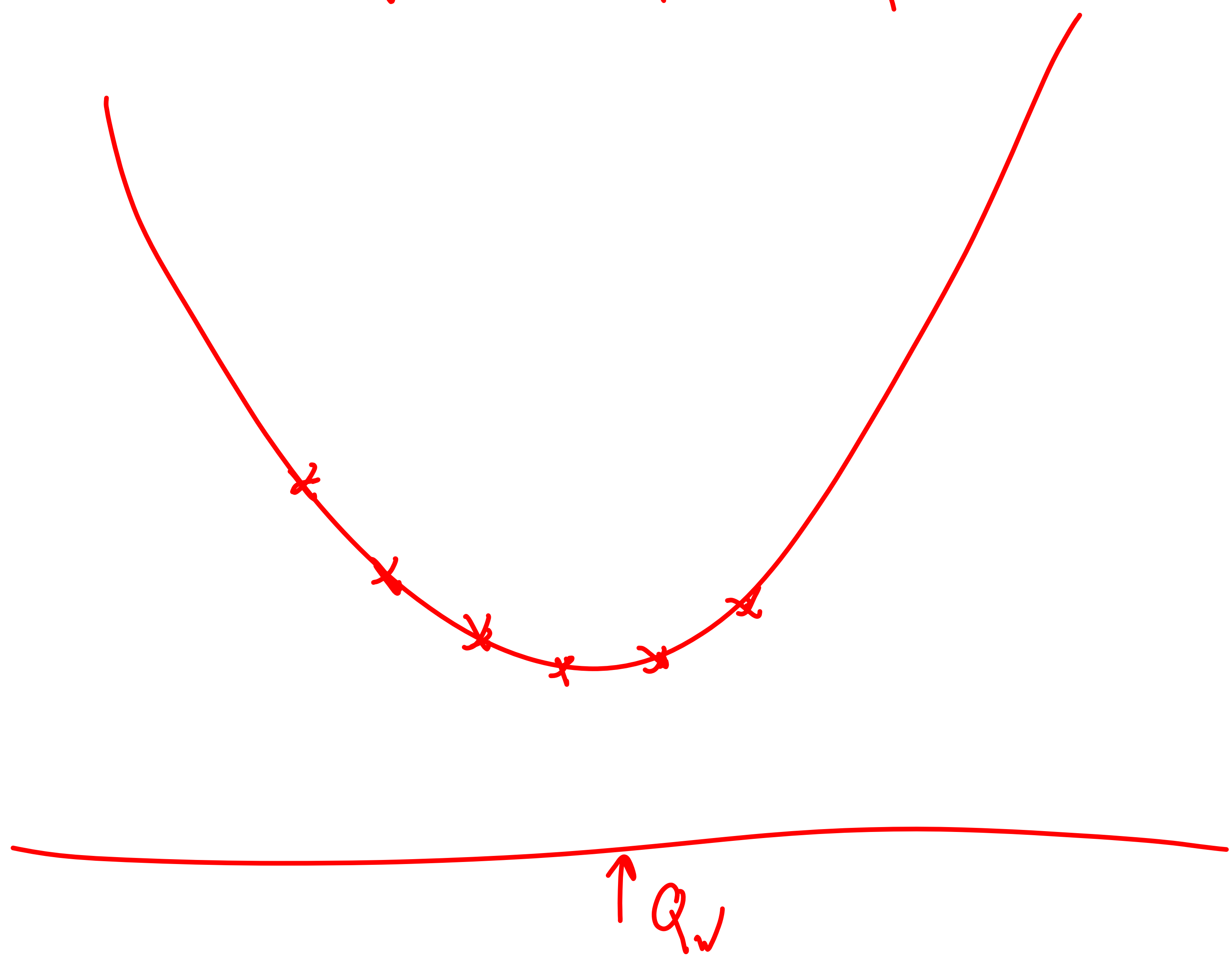
Total Cost =

$$\frac{A\lambda}{Q} + \frac{IQ}{2}$$

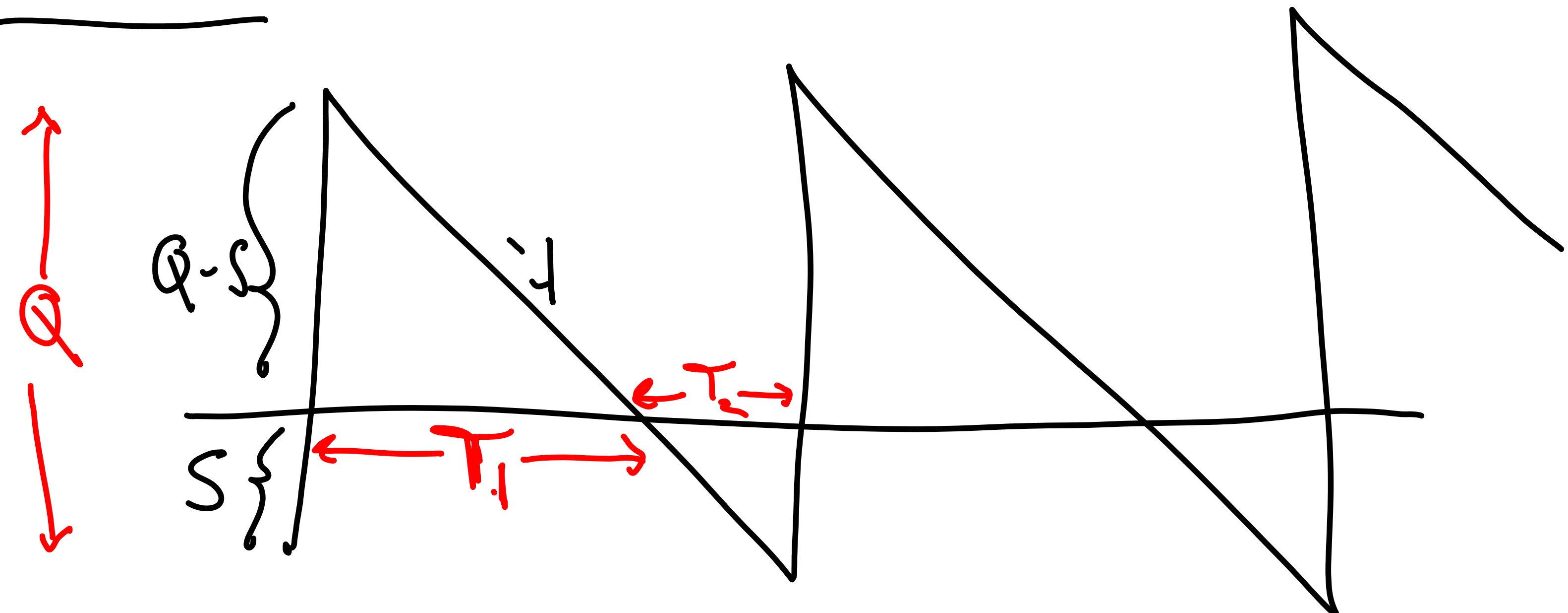
choose Q to minimise

$$-\frac{A\lambda}{Q^2} + \frac{I}{2} = 0 \qquad Q_{Wilson} = \sqrt{\frac{2\lambda A}{I}}$$

Total as function of φ



Model 2



π = penalty per unit of back ordered stuff.

Costs

Ordering $\quad A\lambda/Q$

Inventory $\quad I \times \frac{Q-S}{2} \times \frac{T_1}{T_1+T_2} = \frac{1}{2} I (Q-S) T_1 \lambda / Q$

Penalty $\quad \pi \times \frac{S}{2} \times \frac{T_2}{T_1+T_2} = \frac{1}{2} \pi S T_2 \lambda / Q$

Total Cost =

$$\frac{A\lambda}{Q} + \frac{I(Q-S)T_1\lambda}{2Q} + \frac{\pi S T_2 \lambda}{2Q}$$

$$\lambda T_1 = Q - S$$

$$\lambda T_2 = S$$

Total cost =

$$K = \frac{A\lambda}{Q} + \frac{I(Q-S)^2}{2Q} + \frac{\pi S^2}{2Q}$$

Solve

$$\frac{\partial K}{\partial Q} = \frac{\partial K}{\partial S} = 0$$

$$H = \begin{bmatrix} \frac{\partial^2 K}{\partial Q^2} & \frac{\partial^2 K}{\partial Q \partial S} \\ \frac{\partial^2 K}{\partial Q \partial S} & \frac{\partial^2 K}{\partial S^2} \end{bmatrix}$$ is negative definite.

$$S = (2\lambda A I / \pi(\pi + I))^{\frac{1}{2}}$$

$$Q = Q_w((\pi + I)/\pi)^{\frac{1}{2}}$$

$$K = K_w(\pi/(\pi + I))^{\frac{1}{2}}$$