11\02\09

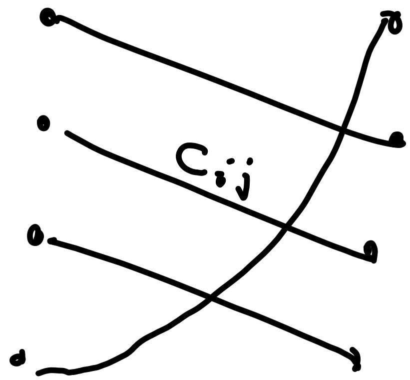
Assignment Problem

n people

n jobs

If person i does job j then the cost is c_{ij}

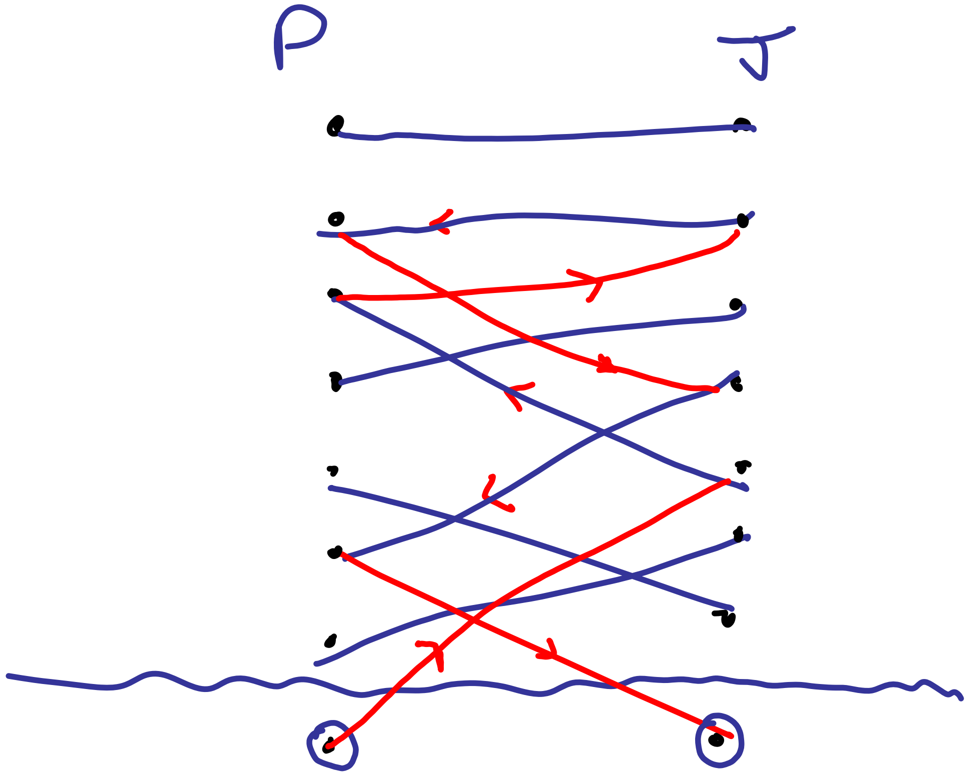
Problem: Find assignment $a : [\text{people}] \longrightarrow [\text{jobs}]$ that minimises $\sum_{i=1}^{n} c_{i, a(i)}$



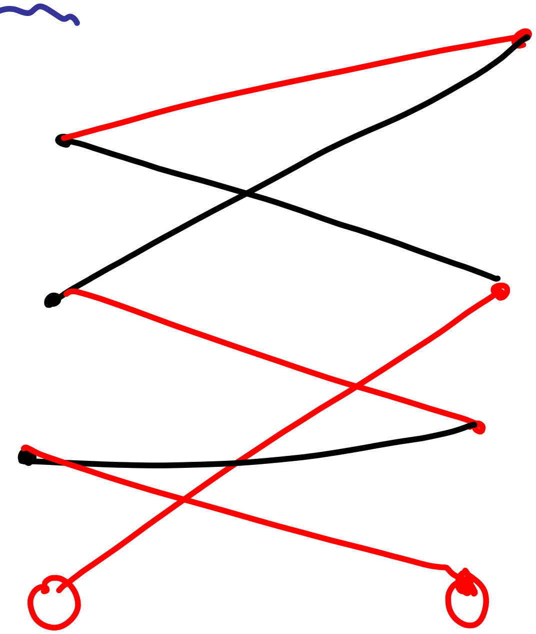
Find minimum cost perfect matching

$$\begin{bmatrix} x & x & x & x \\ x & x & x & x \\ x & x & x & x \\ x & x & x & x \end{bmatrix}$$

Augmenting Path



Want to increase size of matching

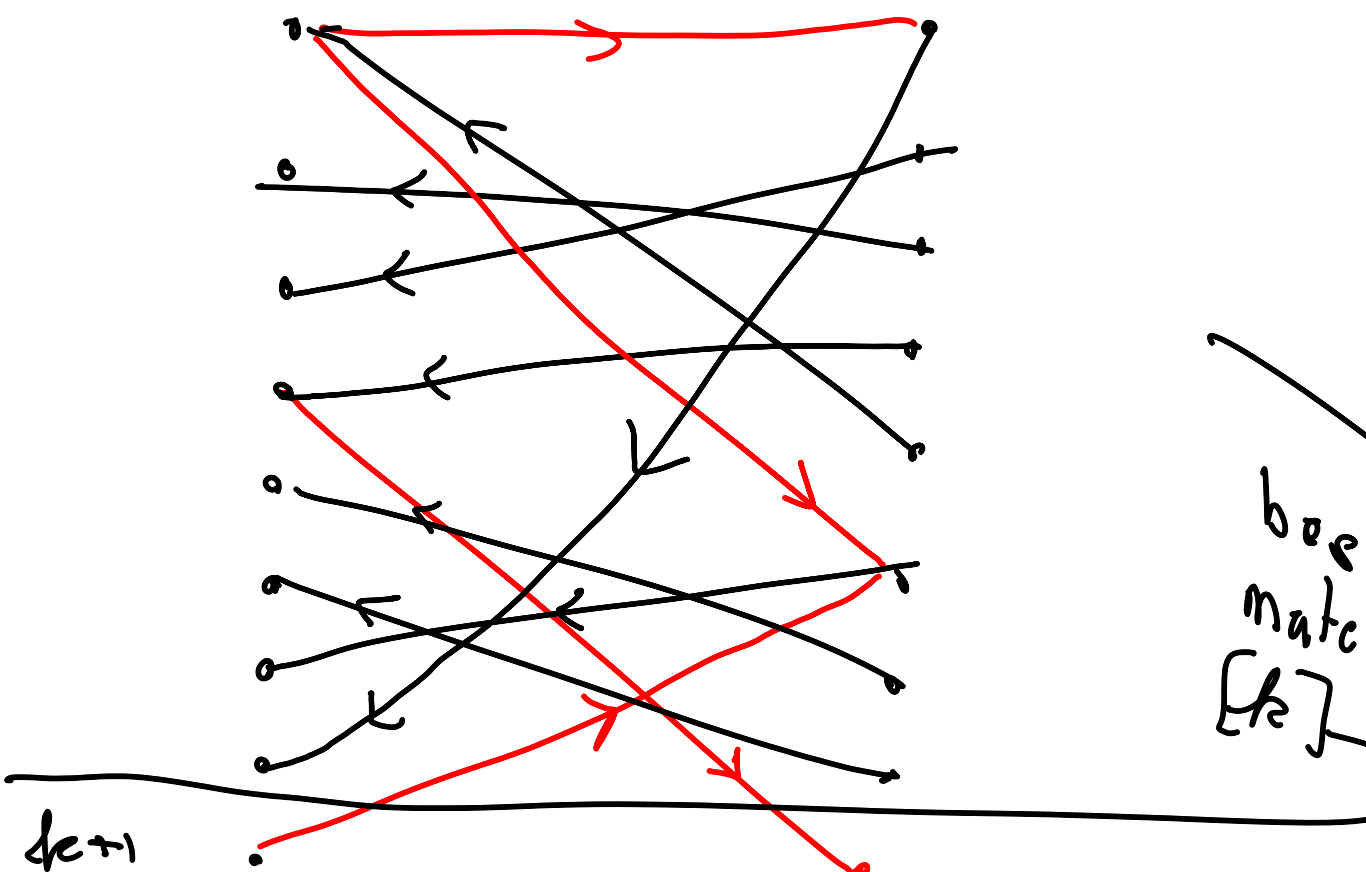


Suppose we have best matching of first k "people" to first k "jobs".

Trivial for $k=1$.

Use shortest path algorithm to find best matching of $k+1$ to $k+1$

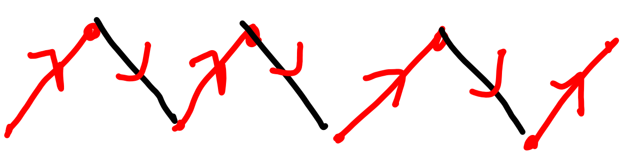
Digraph



best matching

[k] → [k]

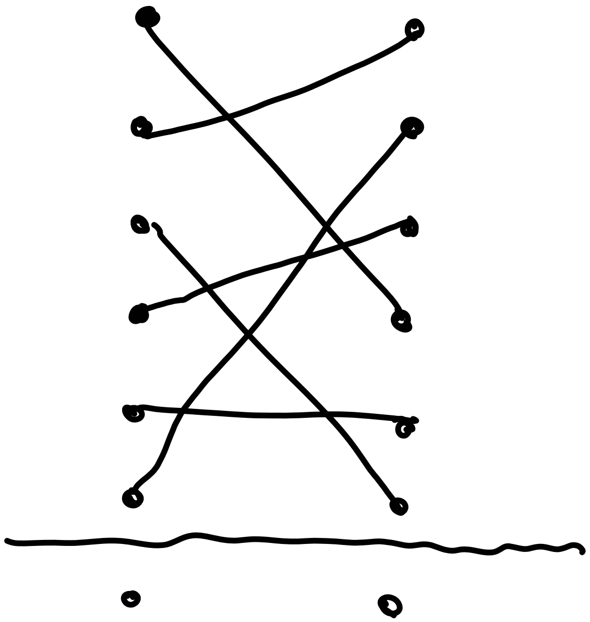
i) Find shortest path

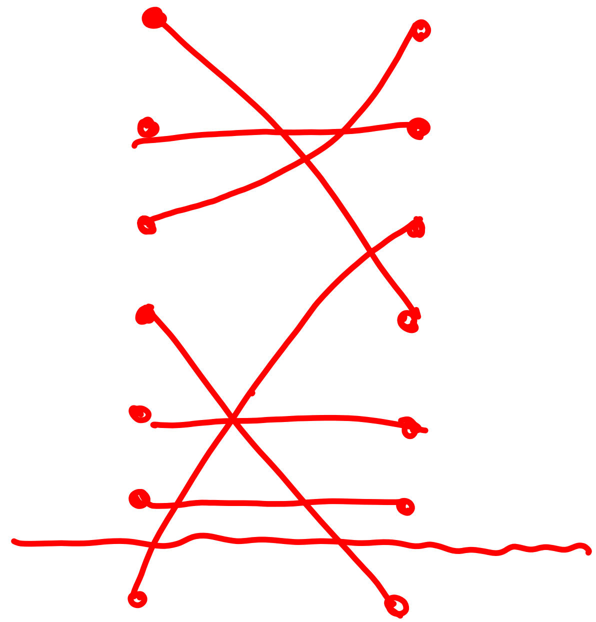


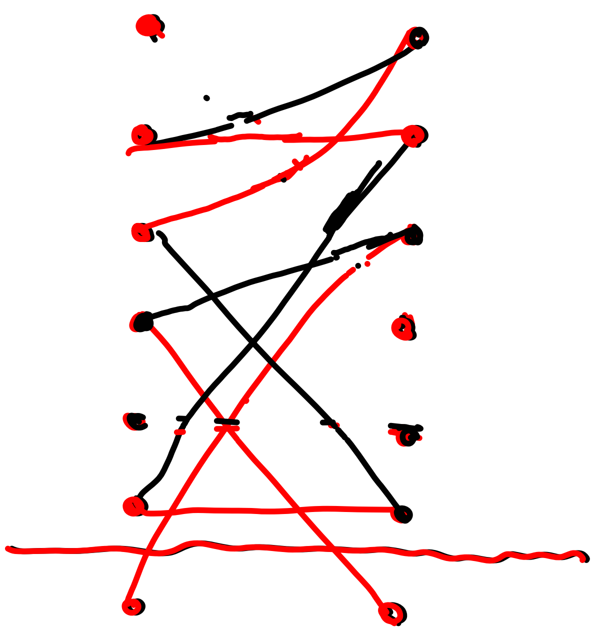
Length

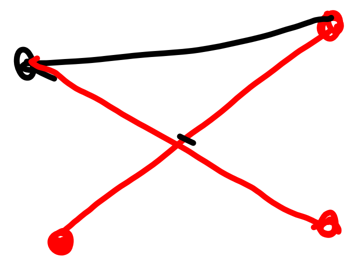
$i \xleftarrow{-c_{ij}} j$

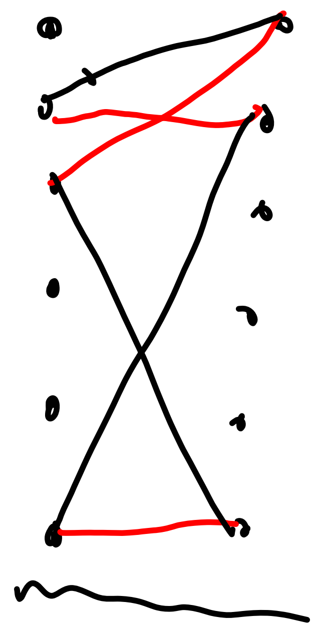
$i \xrightarrow{c_{ij}} j$











Matching M_1 & M_2

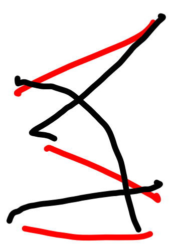
$[k] \to [k]$ $\qquad$ $[k+1] \to [k+1]$

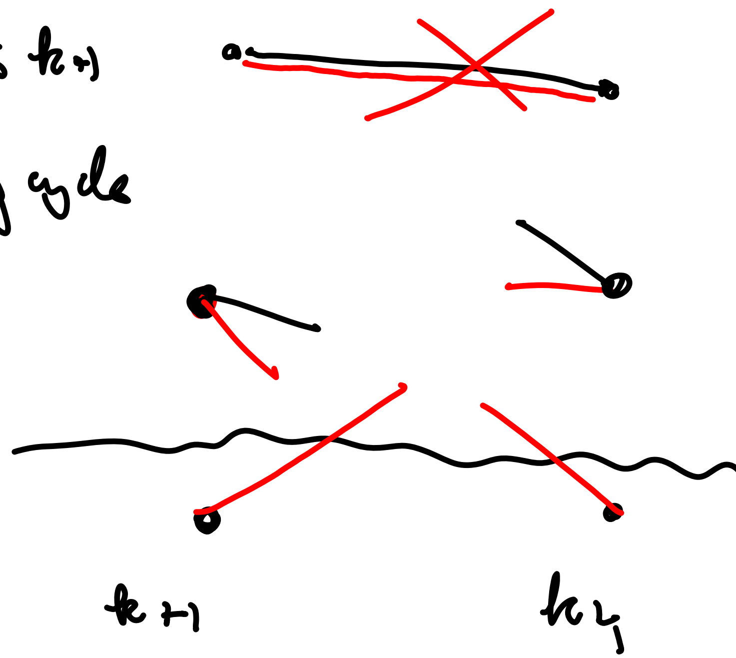
$$M_1 \oplus M_2 = (M_1 \setminus M_2) \cup (M_2 \setminus M_1)$$

= Path P: $k+1$ .. to $k+1$

plus alternating cycle

$C_1, C_2, \dots, C_k$





$$c(M_2) = c(M_1) + c(P) + \sum_{i=1}^{k} c(C_i)$$

Observation: $c(C_i) \geq 0$ shortest path problem is solvable in $O(n^3)$ time.

If $c(C_1) < 0$ then one can "use" it to improve M_1

Not possible M_1 is best $[k] \to [k]$

So $M_2' = M_1 \oplus P$ is at least as good as M_2.

So, I should find best P — shortest $(k+1)$ to $(k+1)$ path.