Note Title 10/28/2009

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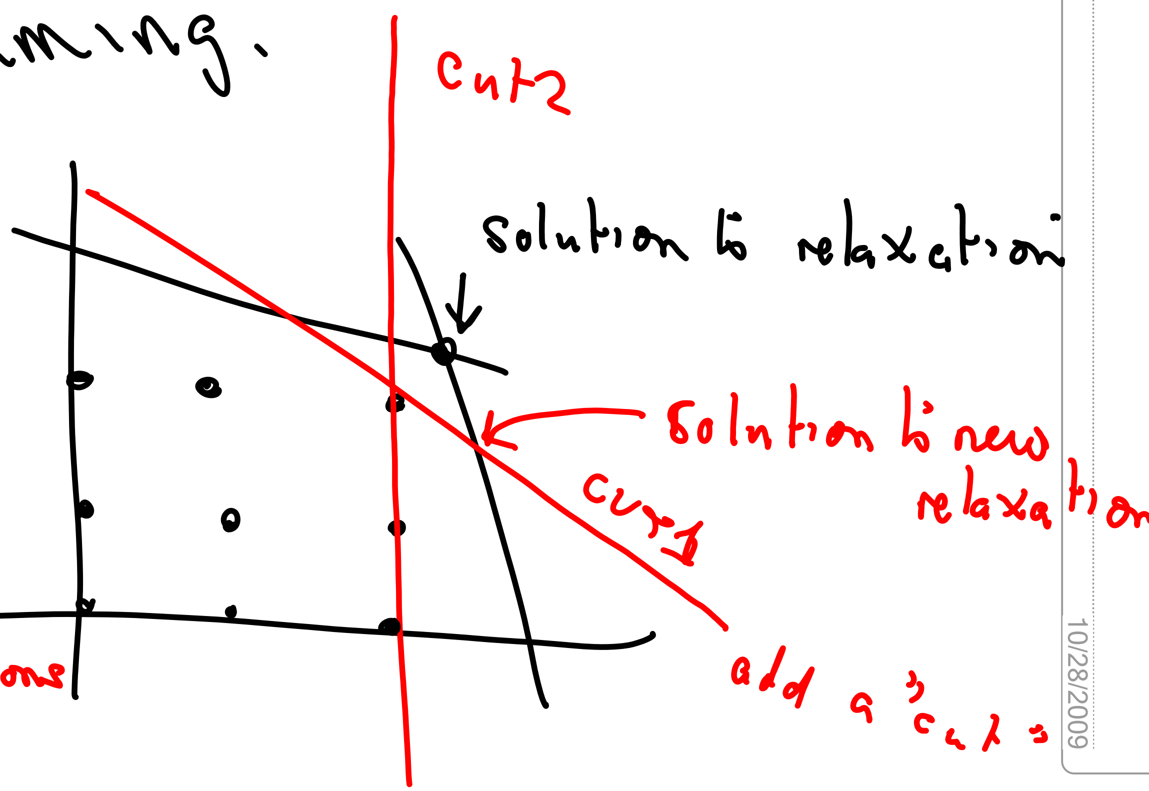
A cutting plane algorithm for pure (all variables are integer) integer programming.

cut

1) not satisfied by current LP solution.

2) is satisfied by all integer solutions



Gomory Cuts

Suppose

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b$$

for some real values $a_1, a_2, \ldots, a_n, b$

[In use, this is a simplex row]

Write $\quad a_i = \lfloor a_i \rfloor + f_i \qquad f_i \geq 0$

$$b = \lfloor b \rfloor + f$$

$6\frac{1}{3} = 6 + \frac{1}{3}$

$-6\frac{1}{3} = -7 + \frac{2}{3}$

and assume that $f > 0$ i.e. b is non-integral.

$$(\lfloor a_1 \rfloor + f_1) x_1 + (\lfloor a_2 \rfloor + f_2) x_2 + \cdots = \lfloor b \rfloor + f$$

$$\sum_{j=1}^{n} f_j x_j - f = \lfloor b \rfloor - \sum_{j=1}^{n} \lfloor a_j \rfloor x_j$$

Suppose that $x_1, x_2, \ldots, x_n$ are integers $\Rightarrow$ is integer

So if $x_1, \ldots, x_n$ are integers then

$\sum_{j=1}^{n} f_j x_j - f$ is an integer

If in addition $x_1, \ldots, x_n \geq 0$ then

$$\sum_{j=1}^{n} f_j x_j - f \geq -f$$

So if $x_1, \ldots, x_n$ are non-negative integer solutions to

$$a_1 x_1 + \cdots + a_n x_n \le b$$

then

$$f_1 x_1 + \cdots + f_n x_n \ge f$$

Suppose we have solved an LP relaxation and the basic solution is not integer.

Some basic variable x_i is not an integer

$$x_i + \sum_{j \text{ non basic}} b_{ij} x_j = b_{i0} \leftarrow \text{not integer}$$

Suppose we have solved an LP relaxation and the basic solution is not integer.
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The integer solutions satisfy

$$\sum_{j \text{ non-basic}} f_j x_j \geq f > 0$$

$x_j = 0$, j non-basic in current solution and new constraint cuts this off.

Maximise

$$x_1 + 4x_2$$

$$\text{s.t. } 2x_1 + 4x_2 \leq 7$$

$$10x_1 + 3x_2 \leq 14$$

$$x_1, x_2 \geq 0 \text{ \& integer.}$$

B.V.	x_1	x_2	x_3	x_4	R
x_0	-1	-4			0
x_3	2	4	1		7
x_4	10	3		1	14

Objective row: $x_0 - x_1 - 4x_2 = 0$

B.V.	x_1	x_2	x_3	x_4	ξ_1	R
x_0	1		1			7
x_2	$\frac{1}{2}$	1	$\frac{1}{4}$			$7/4$
x_4	$17/2$		$-3/4$	1		$35/4$
ξ_1	$-\frac{1}{2}$		$-\frac{1}{4}$		1	$-3/4$

$$\frac{1}{2}x_1 + x_2 + \frac{1}{4}x_3 = 7/4$$

$$\frac{1}{2}x_1 + \frac{1}{4}x_3 \geq 3/4$$

ξ_1 : slack

B.V.	x_1	x_2	x_3	x_4	ξ_1	R
x_0			$\frac{1}{2}$		2	$11/2$
x_2		1			1	1
x_4			-5	1	17	-4
ξ_1	1		$\frac{1}{2}$		-2	$\frac{3}{2}$

B.V.	x_1	x_2	x_3	x_4	ξ_1	R
x_0				$1/10$	$27/10$	$51/10$
x_2		1			1	1
x_3			1	$-1/5$	$-17/5$	$4/5$
x_1	1			$1/10$	$-3/10$	$11/10$
ξ_2				$-1/10$	$-7/10$	$-1/10$

$$x_0 + \frac{1}{10}x_4 + \frac{27}{10}\xi_1 = 51/10$$

$$\frac{1}{10}x_4 + \frac{7}{10}\xi_1 - \xi_2 = \frac{1}{10}$$

B.V.	x_1	x_2	x_3	x_4	ξ_1	ξ_2	R
x_0					3		5
x_2		1			1		1
x_3			1		-1	2	1
x_1	1				-1	1	1
x_4				1	7	-10	1

Solution: $x_1 = x_2 = 1$

$x_0 = 5$