10\23\09

Set Covering

$S_1, S_2, \ldots S_m \subseteq S$

$c_1, c_2, \ldots c_m$ real

$I \subseteq [m]$; $c(I) = \sum_{i \in I} c_i$

I is a <u>cover</u> if $\bigcup_{i \in I} S_i = S$

Problem: find minimum cost cover

"Practical" example

Airline: Timetable = {flight legs}

e.g. Pgh → LA.

S: a "feasible" set of flight legs for one crew for one period.

Pgh → LA → SF → Pgh

Need to "cover" timetable as cheaply as possible

Variables: $x_j = \begin{cases} 1: \text{use } S_j \\ 0: \text{don't} \end{cases}$

Cost: $c_1 x_1 + c_2 x_2 + \cdots + c_m x_m$

Constraints: $i \in \bigcup_{\{j : x_j = 1\}} S_j \quad \forall i$

$a_{ij} = \mathbb{1}_{i \in S_j} = \begin{cases} 1: i \in S_j \\ 0: i \notin S_j \end{cases}$

Indicator Variable

$\sum_j a_{ij} x_j \geq 1 \quad \forall i$

Branch and Bound Algorithm

minimize $\underline{c} \cdot \underline{x}$

$A\underline{x} = b$

$\underline{x} \geq 0$

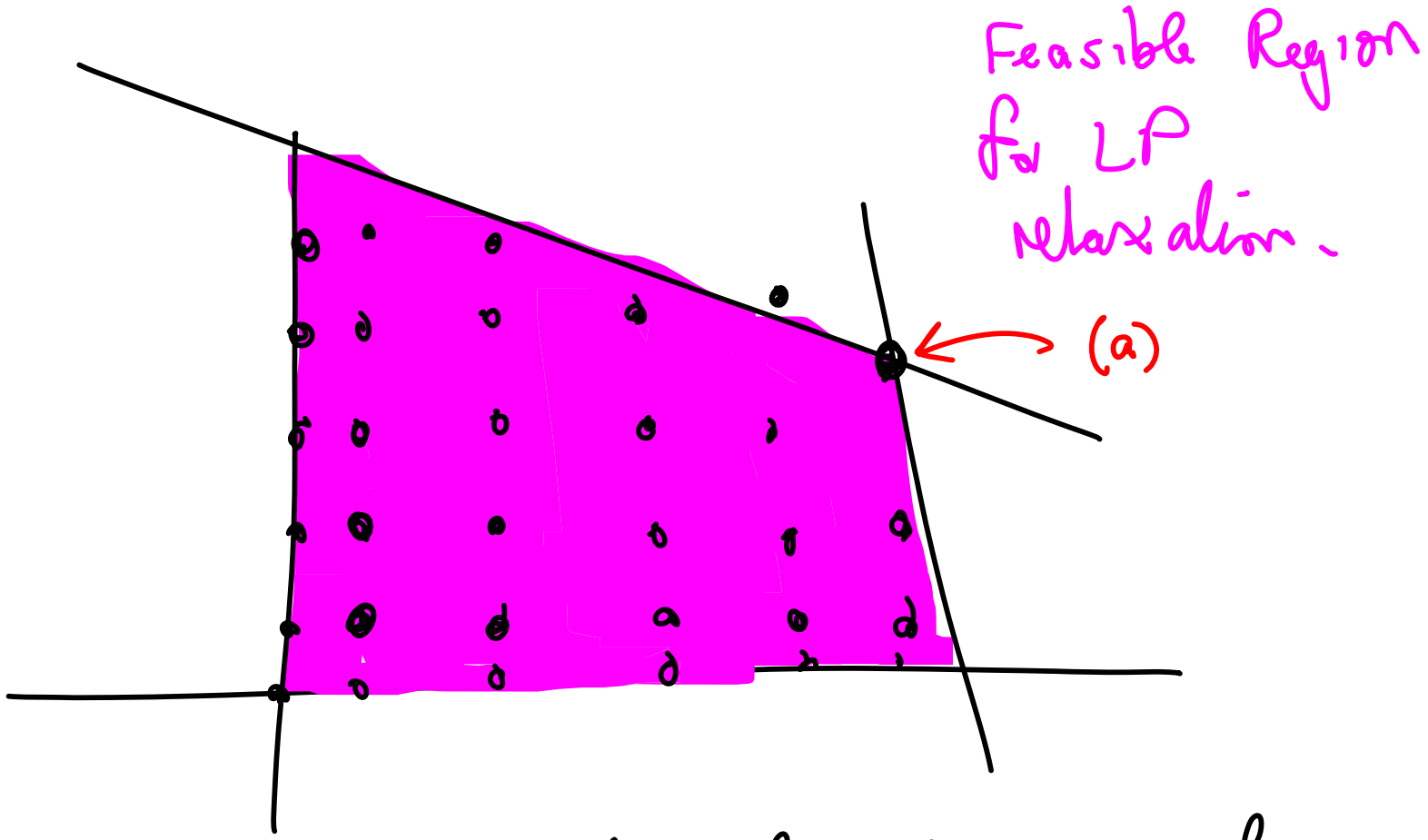
x_j integer $j \in J$

LP relaxation, ignore ↗ .

● ← Solve LP relaxation

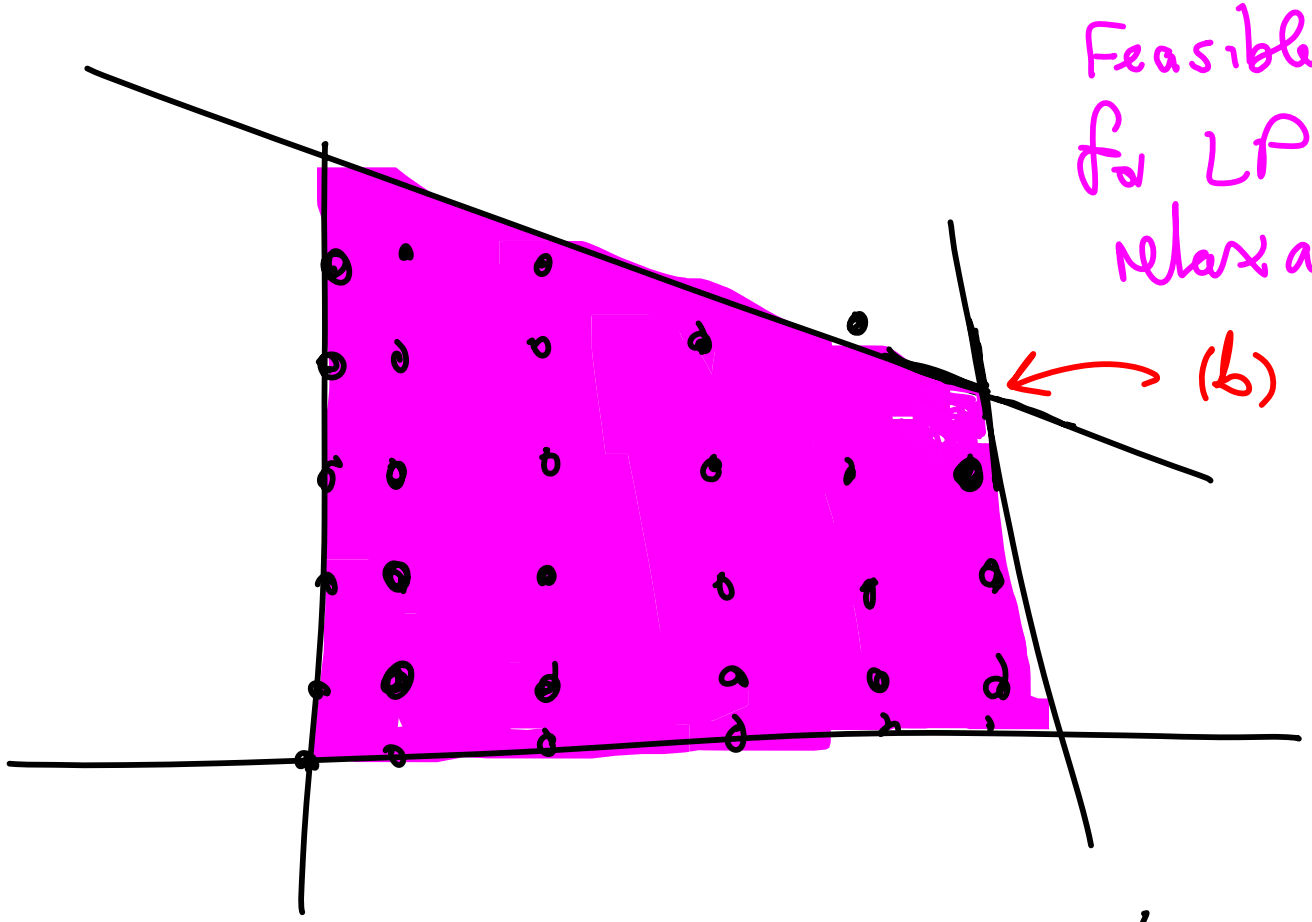
(a) Answer has x_j integer $j \in J$

(b) (a) is not true



- is best one whole feasible region of LP relaxation

$\Rightarrow$ is best feasible integer point

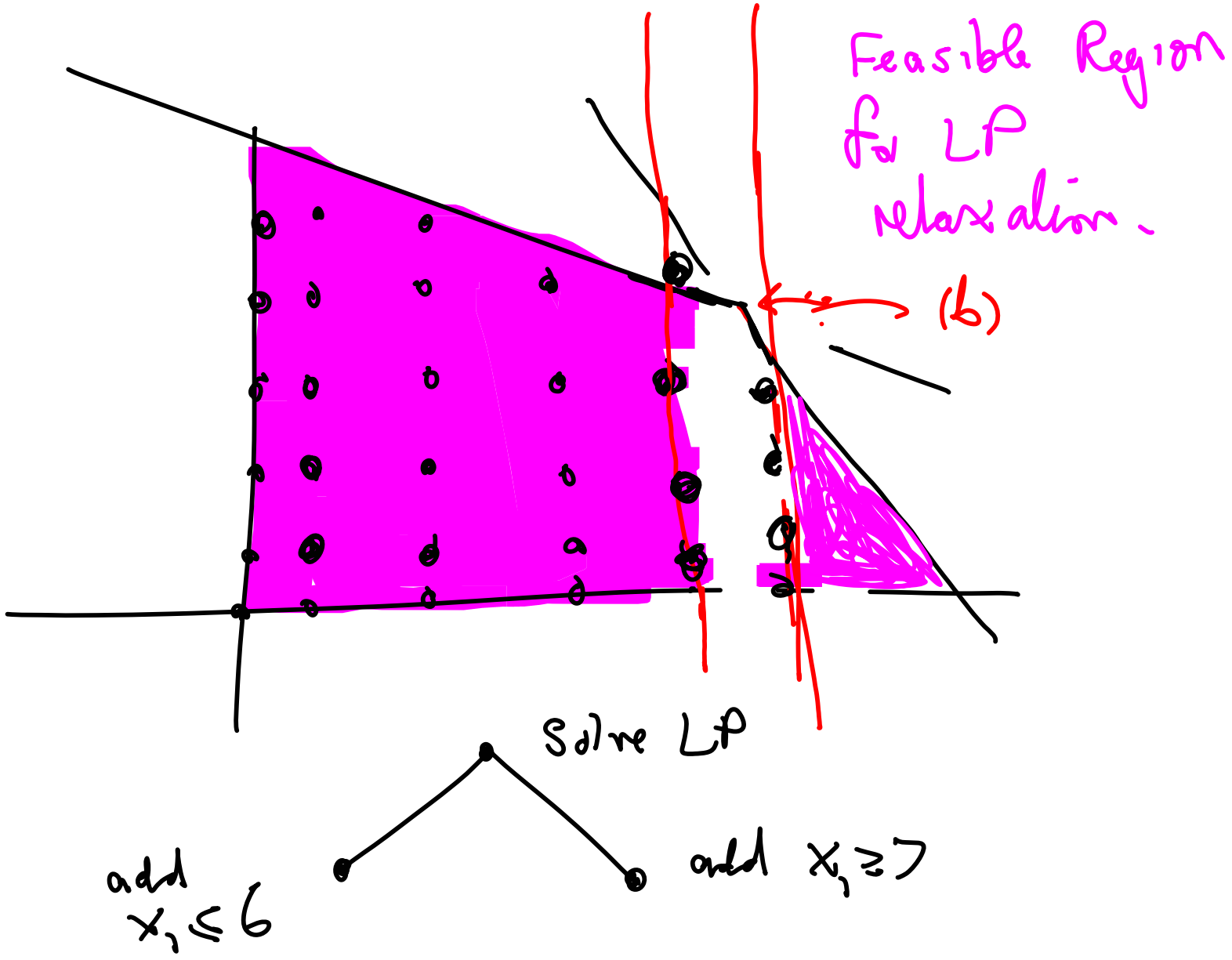


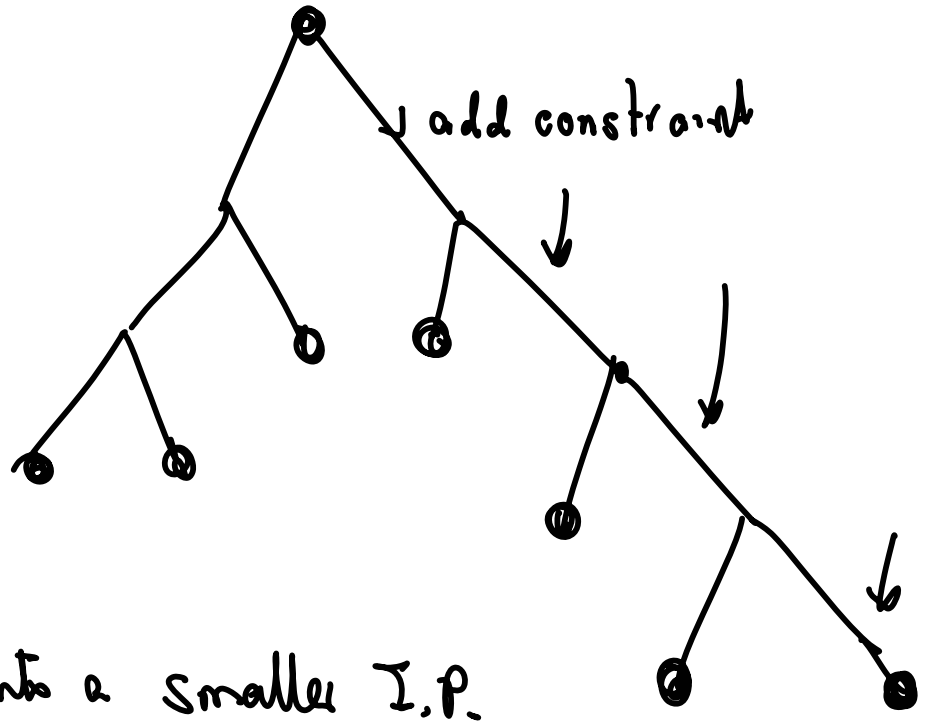
Solution to LP is $x_1 = 6.3$

$x_2 = 4.7$

Split I.P. into two smaller problems.

$x_1 \leq 6$ and $x_1 \geq 7$

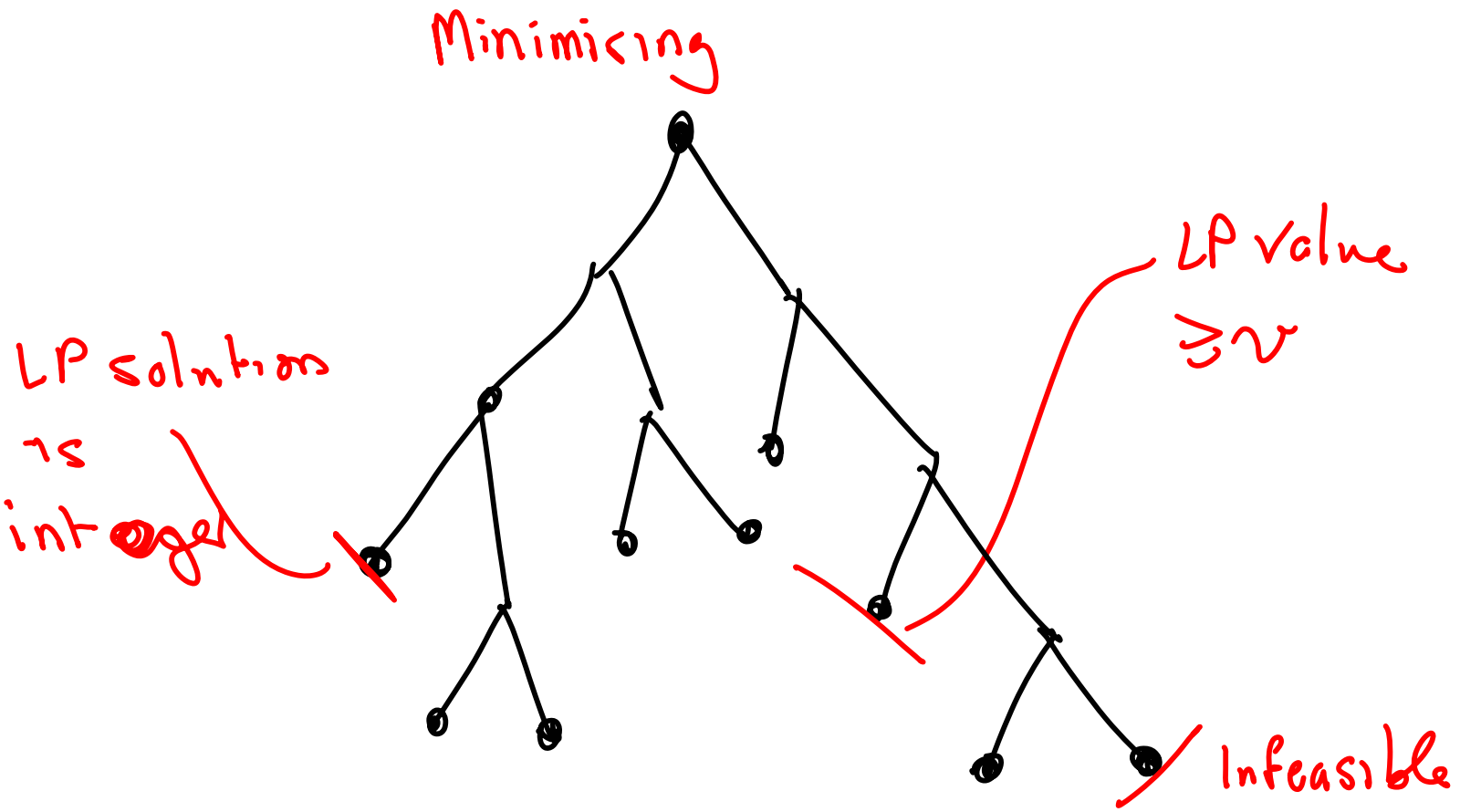

Feasible Region
for LP
relaxation.
(b)
Solve LP
add
$x_1 \leq 6$
add $x_1 \geq 7$



(i) ● → represents a smaller I.P.

(ii) The feasible regions of these contain feasible region of lop.

(iii) Best solution is all ● is optimal.



(iv) If ● is infeasible as LP, need not do any more work on this branch.

(v) Bounding: suppose I have a feasible integer solution of value v.